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Fractional branes in non-compact type IIA orientifolds

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Abstract: We study fractional D-branes in the Type-IIA theory on a non-compact orientifold of the orbifold $\mathbb{C}^3/\mathbb{Z}_3$ in the boundary state formalism. We find that the fractional D0-branes of the orbifold theory become unstable due to the presence of a tachyon, while there is a stable D-instanton whose tachyon gets projected out. We propose that the D-instanton is obtained after tachyon condensation. We evidence this by calculating the Whitehead group of the Abelian category of objects corresponding to the boundary states as being isomorphic to $\mathbb{Z}_2$.

Keywords: Topological Strings, D-branes.

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1. Introduction and summary

D-branes have become a sine qua non for the studies of non-perturbative string theory. The description of D-branes in terms of open strings makes it possible to treat them within the scope of a boundary conformal field theory (BCFT). Since the BCFT does not rely upon spacetime supersymmetry, this formulation is well-suited for treating BPS as well as non-BPS states on the same footing. D-branes have been studied on a variety of singular and non-singular target spaces using BCFT [1 − 24]. D-branes in the Type-II theories on non-compact orbifolds constitute an interesting class of such theories [27 − 29, 1 − 3]. The stable states in the spectrum of such theories have been identified with D-branes wrapped on various supersymmetric cycles of the target space. One of the purposes of the present article is to go beyond orbifold backgrounds to the more interesting orientifold backgrounds [30, 31, 49 − 51] and examine the spectrum of stable D-branes.

While open strings arise as fluctuations of D-branes, the latter can be thought of as a geometric description of the gauge degrees of freedom ensuing from the terminal points of the former. Hence the interpretation of the states in the spectrum of open strings in terms of D-branes becomes transparent when viewed in the closed string channel. This is brought out through boundary states, which incorporate the boundary conditions of open strings in the closed string language. A formulation of BCFT is in terms of such boundary state. We consider D-branes in the Type-IIA string theory on an orientifold of the three-dimensional orbifold $\mathbb{C}^3/\mathbb{Z}_3$ using the boundary state formulation [52, 54, 64]. The orientifold reduces spacetime supersymmetry further compared to the parent orbifolds [3, 6, 4, 65, 32 − 37, 43 − 48], rendering the standard machinery of the $(2, 2)$ theories unavailable. Nevertheless, D-branes on certain non-compact three-dimensional orientifolds have been studied earlier, most which, however, dealt with the Type-IIB string theory [61, 60]. In the present article we consider the Type-IIA theory on the orientifold, which is a cousin of a certain asymmetric orbifold with magnetic fluxes on D-branes, of the Type-IIB theory via T-duality [38], which makes it rather different from the earlier analyses as we shall observe[1].

Prior to orientifolding the stable D-brane configurations are the fractional D0-branes [1, 2]. In the present example the D0-branes are inflicted with tachyonic instabilities after orientifolding. However, we find stable D-instanton configurations in the model which are otherwise absent in the parent orbifold theory. The unavailability of the $(2, 2)$ machinery poses a major hurdle in arriving at a geometric interpretation of these states. Nevertheless, due to the invariance of the boundary states of the D0-brane under the orientifolding operation we can first consider the geometric objects in the parent orbifold theory. Then,

[1] For example, the Type IIA orientifold model we discuss, is not T-dual to the Type-IIB oreintifold on $\mathbb{C}^3/\mathbb{Z}_3$ of [25].

by lifting the orientifold action on these objects as an automorphism squaring up to the identity leads to the geometric entities present in the orientifold theory. We take this approach in this article.

In the parent orbifold theory the D-brane configurations are identified with objects in the derived category of an Abelian category, where the latter can be identified with the category of coherent sheaves on the blown-up orbifold in the large volume limit and with the category of irreducible representation of an associate quiver in the so-called orbifold regime [26]. D-brane configurations in the Type-IIA and Type-IIB theories can be identified as the elements of the K^1 and K^0 groups of the Abelian category and its equivariant descendants on spacetime orbifolds. On orientifolds the stable D-branes are classified by higher K-theoretic charges. In the present case, the D-brane configurations correspond to the elements of the Whitehead group, K^1, of the Abelian category associated with the orientifold space. Assuming that the orientifold operation induces an automorphism on the objects of the Abelian category that squares up to the identity, that is, the boundary states, we calculate the Whitehead group of the Abelian category. We find that the Whitehead group is isomorphic to $\mathbb{Z}_2$, which we identify as the charge of the D-instanton.

The paper is organized as follows. In the following section we describe the orientifold on which we compactify the Type-IIA theory. Then in the two subsequent sections we construct the crosscap state and the D-brane boundary state respectively in the BCFT formulation. Next we calculate various one-loop amplitudes. We show that the D0-branes are plagued with the tachyonic instability and that the D-instanton gives rise to a stable configuration, instead. The following section deals with the K-theoretic analysis. Finally we conclude with discussion of the results. Some of the useful formulas in our notation and conventions have been relegated to the appendix.

2. The orientifold

Let us begin our discussion by describing the theory under consideration. In this section we first discuss the orientifold action and then the spectrum of the massless closed string states. This analysis provides the space-time fields present in the theory.

2.1 Orientifold action

We consider Type–IIA theory in the light-cone gauge on the orientifold [33] $\mathbb{C}^3/\mathcal{G}$ with $\mathcal{G} = (\Omega \cdot \mathcal{R} \cdot (-1)^{F_L}) \otimes G$, where $\mathcal{G}$ is a discrete group with both geometric and non-geometric parts. The first piece of $\mathcal{G}$ in parentheses refers to the diagonal group isomorphic to $\mathbb{Z}_2$ obtained by combining three $\mathbb{Z}_2$ groups. Of these, Ω, isomorphic to $\mathbb{Z}_2$, acts on the world-sheet fields by reversal of parity, $\Omega : \sigma \longmapsto \pi - \sigma$, where σ denotes the spatial coordinate of the world-sheet. The anti-holomorphic involution $\mathcal{R}$, also isomorphic to $\mathbb{Z}_2$, acts by complex conjugation $\mathcal{R} : Z^i \longmapsto \overline{Z}^i$, on the complex bosonic fields of the world-sheet theory, whose zero-modes are identified with the complex coordinates of the $\mathbb{C}^3$. This is tantamount to a reflection of three of the corresponding real coordinates. These are further accompanied with $(-1)^{F_L}$, which changes the sign of the left-moving space-time fermions

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in order for making $\mathcal{G}$ into a symmetry group of the Type–IIA theory. Finally, $\mathcal{G}$ contains a cyclic group G isomorphic to $\mathbb{Z}_N$ and generated by g acting as,

$$g : Z^i \longmapsto e^{2\pi i k v_i} Z^i, \tag{2.1}$$

on the complex coordinates of $\mathbb{C}^3$ and similarly on their fermionic counterparts Ψ^i, $i = 1, 2, 3$. While much of the discussion in the sequel remain unaltered for any odd integer N, we shall restrict ourselves to the specific case of $N = 3$ for simplicity. Thus, we have $k = 0, 1, 2$ and $\vec{v} = (1/3, 1/3, -2/3)$. The compact cousin of this model was discussed in [33]. The combined action of Ω and $(-1)^{F_L}$ on the worldsheet fermions is given by,

$$\Omega \cdot \mathcal{R} : \Psi^i(\sigma) \longmapsto \overline{\widetilde{\Psi}^i}(\pi - \sigma), \tag{2.2}$$

where a tilde designates a right-moving field. Let us also note that by abuse of notation we denote the groups Ω, $\mathcal{R}$ and $(-1)^{F_L}$ as well as their respective generators by the same symbols.

We have described the action of $\mathcal{G}$ on the fields of the theory, which can be used to describe the action on the corresponding oscillators in their mode expansions, as described in appendix. Let us now discuss its lift to the states of the theory. The unique ground state $|\,0\rangle_{\text{NS}}$ in the NS-sector remains unaffected. Let us write the left- and the right-moving Rammond ground states, hereafter referred to as the R-ground states, as $|\,\mathbf{s}\rangle_L = |\,s_0, \vec{s}\rangle_L = |\,s_0, s_1, s_2, s_3\rangle_L$ and $|\widetilde{\mathbf{s}}\rangle_R = \left|\widetilde{s}_0, \vec{\widetilde{s}}\right\rangle_R = |\,\widetilde{s}_0, \widetilde{s}_1, \widetilde{s}_2, \widetilde{s}_3\rangle_R$ respectively, where s_a, $\widetilde{s}_a = \pm\frac{1}{2}$ for $a = 0, 1, 2, 3$. In our convention, the left-moving R-ground states, transforming as $\mathbf{8_s}$ under the Poincaré group $SO(8)$ of the space-time transverse to the light-cone are chosen to be the ones with a even number of $-1/2$'s while the right-moving R-ground states, transforming as $\mathbf{8_c}$ under the $SO(8)$, are taken to be the ones in which an odd number of $-1/2$'s occur. The action of $\mathcal{G}$ on the various R-ground states is given by,

$$\begin{aligned}
g^k : |\,s_0, \vec{s}\rangle_L &\longmapsto e^{2\pi i k \vec{v}\cdot\vec{s}}\, |\,s_0, \vec{s}\rangle_L\,, \quad \left|\widetilde{s}_0, \vec{\widetilde{s}}\right\rangle_R \longmapsto e^{2\pi i k \vec{v}\cdot\vec{\widetilde{s}}} \left|\widetilde{s}_0, \vec{\widetilde{s}}\right\rangle_R; \\
(-1)^{F_L} : |\,s_0, \vec{s}\rangle_L &\longmapsto -\,|\,s_0, \vec{s}\rangle_L\,, \quad \left|\widetilde{s}_0, \vec{\widetilde{s}}\right\rangle_R \longmapsto \left|\widetilde{s}_0, \vec{\widetilde{s}}\right\rangle_R; \\
\Omega \cdot \mathcal{R} : |\,s_0, \vec{s}\rangle_L &\longmapsto |\,s_0, -\vec{s}\rangle_R\,, \quad \left|\widetilde{s}_0, \vec{\widetilde{s}}\right\rangle_R \longmapsto e^{\pi i(\widetilde{s}_0 + \widetilde{s}_1 + \widetilde{s}_2 + \widetilde{s}_3)} \left|\widetilde{s}_0, -\vec{\widetilde{s}}\right\rangle_L = -\left|\widetilde{s}_0, -\vec{\widetilde{s}}\right\rangle_L,
\end{aligned} \tag{2.3}$$

as the sum $\sum_{a=0}^{3} \widetilde{s}_a$ is odd for states belonging to $\mathbf{8_c}$, resulting into

$$\mathfrak{g}(k) \equiv \Omega\cdot\mathcal{R}\cdot(-1)^{F_L}\cdot g^k : |\,s_0, \vec{s}\rangle_L \otimes \left|\widetilde{s}_0, \vec{\widetilde{s}}\right\rangle_R \longmapsto -e^{2\pi i k \vec{v}\cdot(\vec{s}+\vec{\widetilde{s}})} \left|\widetilde{s}_0, -\vec{\widetilde{s}}\right\rangle_L \otimes |\,s_0, -\vec{s}\rangle_R\,, \tag{2.4}$$

after taking into account the minus sign that arises in exchanging fermions. In the above formulas an s in the right-moving state or an $\widetilde{s}$ in the left-moving one is interpreted as their respective numerical values[2].

[2] We can also label the massless states in NS-sectors by their $SO(8)$ weights. Thus, if $s_a = \widetilde{s}_a = \underbrace{(\pm 1, 0, 0, 0)}$, where $\underbrace{\quad}$ denotes all possible permutations, such states belong to $\mathbf{8_v}$ of $SO(8)$. Since $\sum_a s_a = \sum_a \widetilde{s}_a =$ odd holds for a state belonging to $\mathbf{8_v}$, the last equation of (2.3) holds. Thus the action of the orientifold group on massless NS-NS massless states is given by eqn (2.4) but *without* the $-$ sign, since $(-1)^{F_L}$ has no action this time.

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Finally, the left moving and right moving world sheet fermion number operators are defined as

$$\begin{aligned}(-1)^F\,|\,s_0,s_1,s_2,s_3\,\rangle_L &= -e^{\pi i(s_0+s_1+s_2+s_3)}\ \ |\,s_0,s_1,s_2,s_3\,\rangle_L \\ (-1)^{\widetilde{F}}\,|\,\widetilde{s}_0,\widetilde{s}_1,\widetilde{s}_2,\widetilde{s}_3\,\rangle_R &= -e^{\pi i(\widetilde{s}_0+\widetilde{s}_1+\widetilde{s}_2+\widetilde{s}_3)}\ \ |\,\widetilde{s}_0,\widetilde{s}_1,\widetilde{s}_2,\widetilde{s}_3\,\rangle_R\ ,\end{aligned} \tag{2.5}$$

So, we can choose the GSO-projection operators as in the untwisted sector as,

$$\mathcal{P}_U = \begin{cases} \left(\frac{1+(-1)^F}{2}\right)\oplus\left(\frac{1+(-1)^{\tilde{F}}}{2}\right), & \text{in the NS-NS sector} \\ \left(\frac{1-(-1)^F}{2}\right)\oplus\left(\frac{1+(-1)^{\tilde{F}}}{2}\right), & \text{in the R-R sector.}\end{cases} \tag{2.6}$$

In the above expression and in what follows a subscript U is taken to designate a quantity in the untwisted sector, while one in the twisted sector will be designated by T.

2.2 Closed string spectrum

Let us now describe the spectrum of massless closed string states in four dimensions that survive the orientifolding described above.

Untwisted sector

The untwisted sector corresponds to $k = 0$ in equation (2.3). In NS-NS sector, first, we have the four dimensional graviton, $g_{\mu\nu}$ and dilaton, ϕ, whereas the four dimensional part of the B-field is projected out. However, few components of the metric and B-field along the "internal" $\mathbb{C}^3$ directions survive. In terms of the oscillators, these states are given by six $\mathcal{G}$-invariant combinations of states, namely,

$$(\Psi^i_{-\frac{1}{2}}\overline{\widetilde{\Psi}^j}_{-\frac{1}{2}} + \Psi^j_{-\frac{1}{2}}\overline{\widetilde{\Psi}^i}_{-\frac{1}{2}})\,|\,0\,\rangle_{\rm NSNS}\,, \quad i \geq j \tag{2.7}$$

where $i,j = 1,2,3$ and $|\,0\,\rangle_{\rm NSNS}$ denotes the ground state in the NS-NS sector, giving rise to three chiral multiplets in four dimensions[3].

Let us now consider the untwisted states in the R-R sector. From equation (2.4) we see that under $\Omega\cdot\mathcal{R}$ the state $|\,s_0,\vec{s}\rangle$ goes to $\left|\,\widetilde{s}_0,-\vec{\widetilde{s}}\right\rangle$ in the left-moving sector and similarly for the right-moving ones. Thus, the states with $\vec{s}+\vec{\widetilde{s}} = 0$ flip sign under $\mathcal{G}$ and hence go away from the spectrum. However, certain linear combinations of those with $\vec{s}+\vec{\widetilde{s}} \neq 0$, but satisfying $\vec{v}.(s+\widetilde{s}) = 0,1$ survive and can be rearranged in the following seven independent states,

$$\begin{aligned}&|++--\rangle_L\otimes|+++-\rangle_R - |+--+\rangle_L\otimes|+-++\rangle_R\,,\\ &|++--\rangle_L\otimes|++-+\rangle_R - |+-+-\rangle_L\otimes|+-++\rangle_R\,,\\ &|+-+-\rangle_L\otimes|+++-\rangle_R - |+--+\rangle_L\otimes|++-+\rangle_R\,,\\ &|-+-+\rangle_L\otimes|-+--\rangle_R - |--++\rangle_L\otimes|--+-\rangle_R\,,\\ &|-+-+\rangle_L\otimes|---+\rangle_R - |-++-\rangle_L\otimes|--+-\rangle_R\,,\\ &|--++\rangle_L\otimes|---+\rangle_R - |-++-\rangle_L\otimes|-+--\rangle_R\,,\\ &|++++\rangle_L\otimes|-+++\rangle_R - |----\rangle_L\otimes|+---\rangle_R\,.\end{aligned} \tag{2.8}$$

[3] We could have written down these states in terms of the $SO(8)$ weight notation, like we did for R-R case.

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First six of these constitute three chiral multiplets, while the last one joins the four-dimensional dilaton to form one more. Thus, we have four chiral multiplets from the R-R sector in total.

Twisted sector

The massless closed string states in the twisted sector corresponding to $k = 1, 2$ in equation (2.3) are obtained similarly. The GSO-projector in the twisted sector, $\mathcal{P}_T$ is identical to the GSO-projector in untwisted sector, $\mathcal{P}_T = \mathcal{P}_U$. Among the GSO-invariant states a single one from each of the twisted NS-NS and R-R sectors survive the orientifolding. These are

$$\left[\left|0, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right\rangle_L \otimes \left|0, \frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right\rangle_R + \left|0, -\frac{1}{3}, -\frac{1}{3}, -\frac{1}{3}\right\rangle_L \otimes \left|0 - \frac{1}{3} - \frac{1}{3} - \frac{1}{3}\right\rangle_R\right], \tag{2.9}$$

respectively, from the $k = 1, 2$ twisted NS-NS sector and

$$\left[\left|\frac{1}{2}, -\frac{1}{6}, -\frac{1}{6}, -\frac{1}{6}\right\rangle_L \otimes \left|-\frac{1}{2}, -\frac{1}{6}, -\frac{1}{6}, -\frac{1}{6}\right\rangle_R + \left|-\frac{1}{2}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}\right\rangle_L \otimes \left|\frac{1}{2}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6}\right\rangle_R\right] \tag{2.10}$$

respectively, from the $k = 1, 2$ twisted R-R sector. We thus get six chiral multiplets from the untwisted NS-NS sector and three from the untwisted R-R sector, adding up to nine chiral multiplets in total [33]. The remaining state from equation (2.8) pairs up with the the four-dimensional dilaton to form an additional *dilaton multiplet*. Furthermore, the NS-NS and R-R twisted sectors together contribute one chiral multiplet. As there is no vector multiplet, all the D0-branes (including the fractional branes) are projected out by the orientifolding. We shall confirm this from the analysis of the open string states in the following sections.

3. The crosscap state

In order to study the D-branes in the presence of the orientifold plane $O6$ in the model at hand we need to study the open descendants of our model. This involves constructing the crosscap state, corresponding to the orientifold, and the boundary states corresponding to the D-branes. These are the states of open strings in the closed channel, also known as the direct or tree channel.

In this section we construct the crosscap state for the model at hand. The boundary states will be discussed in the following section. A simplification in the construction of the crosscap state in this model ensues from the fact that the amplitudes can be obtained from crosscap states which does not have a twisted sector [32]. In order to illustrate this let us consider the Klein bottle amplitude $\mathcal{K}$. For a general $G \cong \mathbb{Z}_N$ the Klein bottle amplitude is given by

$$\mathcal{K} = \frac{1}{N} \operatorname{Tr}\left[\Omega \cdot \mathcal{R} \cdot (-1)^{F_L} \cdot (1 + g + \cdots + g^{N-1}) \mathcal{P} q^{H_c}\right] \tag{3.1}$$

in the NS-NS or R-R sectors. However, from the action of the group $\mathcal{G}$ given in the previous section we find that the generators satisfy

$$\Theta g^k = g^{N-k} \Theta, \tag{3.2}$$

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where we introduced $\Theta = \Omega \cdot \mathcal{R} \cdot (-1)^{F_L}$ for typographic ease. Now, since the Hamiltonian is invariant under the action of the group G, the energy eigenstates of the system are also eigenstates of the elements of G. Let us fix a mutually orthogonal set of bases of such states, $\{|\,m\rangle \,|m = 0, \ldots, N-1\}$, satisfying

$$g^k\,|\,m\rangle = e^{\frac{2\pi i m k}{N}}\,|\,m\rangle\,, \tag{3.3}$$

for $k = 0, \ldots, N-1$. Hence the trace in equation (3.1) becomes a sum over the expectation values in these states. With these states we derive

$$\langle m\,|\,\Theta g^k \mathcal{P} q^{H_c}\,|\,m\rangle = e^{\frac{2\pi i k m}{N}}\,\langle m\,|\,\Theta \mathcal{P} q^{H_c}\,|\,m\rangle\,, \tag{3.4}$$

and

$$\langle m\,|\,g^{N-k}\Theta \mathcal{P} q^{H_c}\,|\,m\rangle = e^{-\frac{2\pi i k m}{N}}\,\langle m\,|\,\Theta \mathcal{P} q^{H_c}\,|\,m\rangle\,, \tag{3.5}$$

which have to be equal, by (3.2), thereby implying

$$\langle m\,|\,\Theta \mathcal{P} q^{H_c}\,|\,m\rangle \sin\left(\frac{2\pi k m}{N}\right) = 0, \tag{3.6}$$

for non-zero k and m. For $k = 0$ or $m = 0$ this equation collapses to an identity. Consequently, the Klein bottle amplitude (3.1) becomes

$$\begin{aligned} \mathcal{K} &= \frac{1}{N}\sum_{k,m=0}^{N-1} \langle m|\Theta g^k \mathcal{P} q^{H_c}|m\rangle \\ &= \frac{1}{N}\sum_{k,m=0}^{N-1} e^{2\pi i k m/N}\langle m|\Theta \mathcal{P} q^{H_c}|m\rangle, \end{aligned} \tag{3.7}$$

where we used the expression (3.4). Now, in (3.6), the first factor being independent on k, for each non-zero m we can find at least one value of k in the range $0 < k \leq N-1$, such that the sine factor is non-vanishing. Hence, the amplitude $\langle m\,|\,\Theta \mathcal{P} q^{H_c}\,|\,m\rangle$ vanishes for all non-zero m. [4] Thus, finally, the Klein bottle amplitude becomes

$$\begin{aligned} \mathcal{K} &= \frac{1}{N}\sum_{k=0}^{N-1} \langle 0|\Theta \mathcal{P} q^{H_c}|0\rangle \\ &= \langle 0|\Theta \mathcal{P} q^{H_c}|0\rangle. \end{aligned} \tag{3.8}$$

We have thus re-written the Klein bottle amplitude without the g^k-twisted open string states, with no g^k left in the expression. This implies that in the tree channel the Klein bottle amplitude can be obtained from the untwisted crosscap state alone [32, 33, 33]. However, as we shall discuss in the following section, the D-brane boundary states in the closed string picture do contain twisted pieces.

[4] We thank the referee for raising this point.

Let us now write down the equations satisfied by the crosscap state [52 − 54]. These are obtained by twisting the periodic boundary conditions of the closed string fields by the orbifold as well as orientifold action as,

$$\begin{aligned}
&(X^M(\sigma) - \mathfrak{g}(k)X^M(\sigma+\pi)))\,|\,C_6;\eta\,\rangle = 0,\\
&(\partial_\tau X^M(\sigma) + \mathfrak{g}(k)\partial_\tau X^M(\sigma+\pi))\,|\,C_6;\eta\,\rangle = 0,\\
&(\psi^M(\sigma) + i\eta\,\mathfrak{g}(k)\widetilde{\psi}^M(\sigma+\pi,0))\,|\,C_6;\eta\,\rangle = 0,
\end{aligned} \tag{3.9}$$

valid at $\tau = 0$, where $M = 0,\ldots,9$, $\mathfrak{g}(k)$ is as defined in equation (2.4) and C_6 designates the $O6$-plane. Such a crosscap state was first discussed in [39, 40] [5]. The crosscap state is labeled by the spin structure η. The above equations are valid both in the NS-NS and the R-R sector.

In order to obtain the crosscap state as a solution to the equations (3.9) it is convenient to rewrite these equations in terms of the oscillators. In both the NS-NS and the R-R sectors these equations lead to

$$(z_0^i - e^{2\pi ikv_i}\overline{z}_0^i)\,|\,C_6;p,\eta\,\rangle = 0, \tag{3.10}$$

$$(\overline{p}^i + e^{-2\pi ikv_i}p^i)\,|\,C_6;p,\eta\,\rangle = 0, \tag{3.11}$$

$$(\alpha_n^\mu + e^{i\pi n}\widetilde{\alpha}_{-n}^\mu)\,|\,C_6;p,\eta\,\rangle = 0, \tag{3.12}$$

$$(\alpha_l^i + e^{-i\pi l}e^{2\pi ikv_i}\overline{\widetilde{\alpha}^i}_{-l})\,|\,C_6;p,\eta\,\rangle = 0,\quad (\overline{\alpha^i}_l + e^{-i\pi l}e^{-2\pi ikv_i}\widetilde{\alpha}_{-l}^i)\,|\,C_6;p,\eta\,\rangle = 0, \tag{3.13}$$

in terms of the bosonic coordinates, momenta and oscillators described in the appendix. Similarly, the equations for the fermionic oscillators are,

$$(\psi_r^\mu + i\eta e^{-i\pi r}\widetilde{\psi}_{-r}^\mu)\,|\,C_6;p,\eta\,\rangle = 0, \tag{3.14}$$

$$(\Psi_r^i + i\eta e^{-i\pi r}e^{2\pi ikv_i}\overline{\widetilde{\Psi}^i}_{-r})\,|\,C_6;p,\eta\,\rangle = 0,\quad (\overline{\Psi^i}_r + i\eta e^{-i\pi r}e^{-2\pi ikv_i}\widetilde{\Psi}_{-r}^i)\,|\,C_6;p,\eta\,\rangle = 0, \tag{3.15}$$

where p is used to designate the complex momenta p^i in the three internal directions of the $\mathbb{C}^3$ and their complex conjugates, $\overline{p}^i$, with $i = 1,2,3$ and $\eta = \pm 1$. Equations (3.10) – (3.15) can be solved to yield a coherent state for the crosscap as,

$$\begin{aligned}
|\,C_6;p,\eta\,\rangle = \exp\Big(&-\sum_{\substack{\mu=0,3\\ l\in\mathbb{Z}}}\frac{1}{n}(-1)^n\alpha_{-n}^\mu\widetilde{\alpha}_{-n}^\mu - i\eta\sum_{\substack{\mu=0,3\\ r>0}}e^{-i\pi r}\psi_{-r}^\mu\widetilde{\psi}_{-r}^\mu\\
&-\sum_{\substack{i=1,2,3\\ l\in\mathbb{Z}_+}}\frac{e^{i\pi l}}{l}\big(e^{-2\pi ikv_i}\alpha_{-l}^i\widetilde{\alpha}_{-l}^i + e^{-i\pi l}e^{2\pi ikv_i}\overline{\alpha^i}_{-l}\overline{\widetilde{\alpha}^i}_{-l}\big)\\
&-i\eta\sum_{\substack{i=1,2,3\\ r>0}}e^{-i\pi r}\big(e^{-2\pi ikv_i}\Psi_{-r}^i\widetilde{\Psi}_{-r}^i + e^{2\pi ikv_i}\overline{\Psi^i}_{-r}\overline{\widetilde{\Psi}^i}_{-r}\big)\Big)\,|\,C_6;p,\eta\,\rangle^{(0)}
\end{aligned} \tag{3.16}$$

[5] For covariant formulation of crosscaps in Type I strings, see [53, 55] and for crosscaps in asymmetric orientifold theory, see [56].

where $|C_6, p, \eta\rangle^{(0)}$ denotes the Fock space ground state which is unique in the NS-NS sector and is independent of η. The ground state is, however, degenerate in the R-R sector and depends on η. Hence in considering the GSO projection and the orientifolding it will be convenient to treat the NS-NS and R-R sectors separately.

NS-NS sector

Let us first discuss the projections on the crosscap state in the NS-NS sector to obtain the invariant state.

3.0.1 GSO Projection

The NS-NS vacuum is chosen to be odd under the GSO projection. The form of the GSO projectors written in (2.6) are deduced from

$$\begin{aligned}(-1)^F &= -(-1)^{\sum_{r\in\mathbb{Z}+\frac{1}{2}}\left[\sum_\mu \psi^\mu_{-r}\psi^\mu_r+\sum_{i=1}^3\left(\Psi^i_{-r}\Psi^i_r+\overline{\Psi^i}_{-r}\overline{\Psi^i}_r\right)\right]}, \\ (-1)^{\widetilde{F}} &= -(-1)^{\sum_{r\in\mathbb{Z}+\frac{1}{2}}\left[\sum_\mu \widetilde{\psi}^\mu_{-r}\widetilde{\psi}^\mu_r+\sum_{i=1}^3\left(\widetilde{\Psi}^i_{-r}\widetilde{\Psi}^i_r+\overline{\widetilde{\Psi}^i}_{-r}\overline{\widetilde{\Psi}^i}_r\right)\right]},\end{aligned} \tag{3.17}$$

in terms of the oscillators in the left- and right-moving sectors. Their action on the ground state is given by

$$(-1)^F\,|p\rangle^{(0)}_{\text{NSNS}} = (-1)^{\widetilde{F}}\,|p\rangle^{(0)}_{\text{NSNS}} = -\,|p\rangle^{(0)}_{\text{NSNS}}\,, \tag{3.18}$$

where we refrain from mentioning the spin structure explicitly, since the ground state does not depend on it. The above equation implies

$$(-1)^F\,|C_6; p, \eta\rangle_{\text{NSNS}} = (-1)^{\widetilde{F}}\,|C_6; p, \eta\rangle_{\text{NSNS}} = -\,|C_6; p, -\eta\rangle_{\text{NSNS}}\,. \tag{3.19}$$

Thus the GSO-invariant state in the NS-NS sector is,

$$|C_6; p\rangle_{\text{NSNS}} = \frac{1}{\sqrt{3}}\frac{\mathcal{N}_C^{\text{NSNS}}}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}\Big[|C_6; p, +\rangle_{\text{NSNS}} - |C_6; p, -\rangle_{\text{NSNS}}\Big], \tag{3.20}$$

where $\mathcal{N}_C^{\text{NSNS}}$ is the normalization of the NS-NS part of the crosscap. We have separated out a factor of $\frac{1}{\sqrt{2}}$ from the normalization factor to make sure that it correctly reproduces the projector due to orientifold in the open string channel. The second factor, as usual, is for generating the correct GSO projector in the open channel. The NS-NS part of the spatially localized crosscap is obtained by integrating this invariant state over the internal momenta subject to the constraints (3.11). Thus, the position eigenstate corresponding to crosscap in the untwisted NS-NS sector is

$$|C_6\rangle_{\text{NSNS}} = \int\prod_{i=1}^{3} dp^i d\overline{p}^i \delta(p^i + e^{2\pi i k v_i}\overline{p}^i)\,\delta(z_0^i - e^{2\pi i k v_i}\overline{z}_0^i)\,|C_6; p\rangle_{\text{NSNS}} \tag{3.21}$$

3.0.2 Orientifolding

Considering the orientifolding on the crosscap state constructed above, let us first note that $(-1)^{F_L}$ acts trivially on the NS-NS ground state $|p\rangle^{(0)}_{\text{NSNS}}$, which is a spacetime scalar.

The momenta and the coordinates in the internal directions transform under orientifolding as

$$p^i \longmapsto e^{2\pi i k v_i}\overline{p}^i, \quad \overline{p}^i \longmapsto e^{-2\pi i k v_i} p^i, \quad z_0^i \longmapsto e^{2\pi i k v_i}\overline{z_0^i}, \quad \overline{z_0^i} \longmapsto e^{-2\pi i k v_i} z_0^i. \tag{3.22}$$

Since the vertex operator for the NS-NS ground state carrying momenta only along the internal directions

$$e^{i\sum_{m=4}^{9} P_m X^m} = e^{\frac{i}{2}\sum_{i=1}^{3}(\overline{p}^i Z^i + p^i \overline{Z^i})} \tag{3.23}$$

is invariant, the NS-NS ground state $|p\rangle_{\rm NSNS}^{(0)}$ is invariant under the orientifolding. Further, the oscillators transform as

$$\begin{gathered}
\alpha_n^\mu \longmapsto e^{i\pi n}\widetilde{\alpha}_n^\mu, \quad \widetilde{\alpha}_n^\mu \longmapsto e^{-i\pi n}\alpha_n^\mu, \quad \psi_r^\mu \longmapsto e^{i\pi r}\widetilde{\psi}_r^\mu, \quad \widetilde{\psi}_r^\mu \longmapsto -e^{-i\pi r}\psi_r^\mu, \\
\alpha_l^i \longmapsto e^{i\pi l} e^{2\pi i k v_i}\overline{\widetilde{\alpha}_l^i}, \; \overline{\alpha_l^i} \longmapsto e^{i\pi l} e^{-2\pi i k v_i}\widetilde{\alpha}_l^i, \; \widetilde{\alpha}_l^i \longmapsto e^{-i\pi l} e^{2\pi i k v_i}\overline{\alpha_l^i}, \; \overline{\widetilde{\alpha}_l^i} \longmapsto e^{-i\pi l} e^{-2\pi i k v_i}\alpha_l^i, \\
\Psi_r^i \longmapsto e^{i\pi r} e^{2\pi i k v_i}\overline{\widetilde{\Psi}_r^i}, \quad \overline{\Psi_r^i} \longmapsto e^{i\pi r} e^{-2\pi i k v_i}\widetilde{\Psi}_r^i, \quad \widetilde{\Psi}_r^i \longmapsto -e^{-i\pi r} e^{2\pi i k v_i}\overline{\Psi_r^i}, \\
\overline{\widetilde{\Psi}_r^i} \longmapsto -e^{-i\pi r} e^{-2\pi i k v_i}\Psi_r^i.
\end{gathered} \tag{3.24}$$

under orientifolding. The exponential factor in equation (3.16) is invariant under $\mathfrak{g}(k)$. Hence the state $|C_6; p, \eta\rangle_{\rm NSNS,U}$ in equation (3.16) is invariant under the orientifold group. The measure as well as the delta-function in (3.21) is invariant under $\mathfrak{g}(k)$. Hence the crosscap state in NS-NS untwisted sector obtained in equation (3.21) is invariant under $\mathcal{G}$.

R-R sector

Let us now turn to the R-R sector. Unlike its NS-NS counterpart, the ground state in the untwisted R-R sector is degenerate. Let us first discuss these degenerate states. It is convenient to write the zero mode operators in the creation-annihilation basis of the $so(8)$ Clifford algebra.

$$\Gamma^{0,\pm} = \frac{1}{\sqrt{2}}(\psi_0^0 \pm i\psi_0^3), \qquad \Gamma^{i,\pm} = \frac{1}{\sqrt{2}}(\psi_0^{2i+2} \pm i\psi_0^{2i+3}), \quad i = 1,2,3 \tag{3.25}$$

for the left-moving states and similarly for right-moving ones, mutatis mutandis. These satisfy the anti-commutation relations,

$$\{\Gamma^{a,+}, \Gamma^{b,-}\} = \delta^{ab}, \quad \{\Gamma^{a,\pm}, \Gamma^{b,\pm}\} = 0, \tag{3.26}$$

where $a, b = 0,1,2,3$. The untwisted R-R ground states are defined in terms of these operators as

$$(\Gamma^{0,-} + i\eta\widetilde{\Gamma}^{0,-})\,|p,\eta\rangle_{\rm RR}^{(0)} = 0, \quad (\Gamma^{i,-} + i\eta\widetilde{\Gamma}^{i,+})\,|p,\eta\rangle_{\rm RR}^{(0)} = 0. \tag{3.27}$$

The ground states may also be chosen so that all the signs in $\Gamma^\pm$ in this equation are reversed. However, the present choice is the one that is in harmony with the corresponding equations for the non-zero modes, (3.14)–(3.15). The R-R ground state is chosen to be

$$\begin{aligned}
|C_6; p, \eta\rangle_{\rm RR}^{(0)} =& \exp\Big[-i\eta\big(\Gamma^{0,+}\widetilde{\Gamma}^{0,-} + \sum_i \Gamma^{i,+}\widetilde{\Gamma}^{i,+}\big)\Big]\,|----\rangle_L \otimes |+---\rangle_R \\
&- \exp\Big[-i\eta\big(\Gamma^{0,-}\widetilde{\Gamma}^{0,+} + \sum_i \Gamma^{i,-}\widetilde{\Gamma}^{i,-}\big)\Big]\,|++++\rangle_L \otimes |-+++\rangle_R,
\end{aligned} \tag{3.28}$$

where $|\pm,\pm,\pm,\pm\rangle_{L,R}$ are as defined in § 2.1 and

$$\Gamma^{a,\pm}\,|\pm,\pm,\pm,\pm\rangle_L = \widetilde{\Gamma}^{a,\pm}\,|\pm,\pm,\pm,\pm\rangle_R = 0. \tag{3.29}$$

Let us now discuss the GSO and the orientifold projections on the ground state and verify that it is invariant under these operations.

3.0.3 GSO Projection

In the R-R sector the GSO operator (2.6) acts as the chirality operator on the zero modes, namely,

$$(-1)^F_\circ = \Gamma_{11} = \prod_{0,3,\ldots,9} \sqrt{2}\psi_0^M \tag{3.30}$$

in the left moving sector and

$$(-1)^{\widetilde{F}}_\circ = \widetilde{\Gamma}_{11} = \prod_{0,3,\ldots,9} \sqrt{2}\widetilde{\psi}_0^M \tag{3.31}$$

in the right-moving sector. Their respective actions on the ground states, therefore, are given as

$$(-1)^F_\circ\,|C_6;p,\eta\rangle^{(0)}_{\rm RR} = -\,|C_6;p,-\eta\rangle^{(0)}_{\rm RR}\,,, \quad (-1)^{\widetilde{F}}_\circ\,|C_6;p,\eta\rangle^{(0)}_{\rm RR} = |C_6;p,-\eta\rangle^{(0)}_{\rm RR}\,. \tag{3.32}$$

The form of the GSO operator on the non-zero modes becomes

$$\begin{aligned}(-1)^F_\bullet &= (-1)^{\sum_{r\in\mathbb{Z}\setminus\{0\}}\left[\sum_\mu \psi^\mu_{-r}\psi^\mu_r+\sum_{i=1}^3(\Psi^i_{-r}\Psi^i_r+\overline{\Psi^i}_{-r}\overline{\Psi^i}_r)\right]},\\ (-1)^{\widetilde{F}}_\bullet &= (-1)^{\sum_{r\in\mathbb{Z}\setminus\{0\}}\left[\sum_\mu \widetilde{\psi}^\mu_{-r}\widetilde{\psi}^\mu_r+\sum_{i=1}^3(\widetilde{\Psi}^i_{-r}\widetilde{\Psi}^i_r+\overline{\widetilde{\Psi}^i}_{-r}\overline{\widetilde{\Psi}^i}_r)\right]}.\end{aligned} \tag{3.33}$$

They leave the ground states unaltered. Moreover, since the coherent state in equation (3.16) contains an odd number of fermion oscillators from the non-zero mode sector, $(-1)^F_\bullet$ and $(-1)^{\widetilde{F}}_\bullet$ act only by flipping the sign of η in the exponential. The total GSO projection operator in the R-R sector, obtained by combining the two parts, namely,

$$(-1)^F = (-1)^F_\circ\,(-1)^F_\bullet, \quad (-1)^{\widetilde{F}} = (-1)^{\widetilde{F}}_\circ\,(-1)^{\widetilde{F}}_\bullet, \tag{3.34}$$

act on the coherent states as,

$$(-1)^F\,|C_6;p,\eta\rangle_{\rm RR} = -\,|C_6;p,-\eta\rangle_{\rm RR}\,, \quad (-1)^{\widetilde{F}}\,|C_6;p,\eta\rangle_{\rm RR} = |C_6;p,-\eta\rangle_{\rm RR}\,. \tag{3.35}$$

Therefore, the GSO-invariant state in the R-R sector is given by,

$$|C_6;p\rangle_{\rm RR} = \frac{1}{\sqrt{3}}\frac{\mathcal{N}^{\rm RR}}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}\Big[\,|C_6;p,+\rangle_{\rm RR} + |C_6;p,-\rangle_{\rm RR}\Big], \tag{3.36}$$

where $\mathcal{N}_C^{\rm RR}$ is the normalization of the R-R part of the crosscap.

3.0.4 Orientifolding

The action of the orientifold group on the R-R ground states are given in (2.3). The corresponding action on the Γ-matrices is given by

$$\Gamma^{0,\pm} \longmapsto \widetilde{\Gamma}^{0,\pm}, \quad \widetilde{\Gamma}^{0,\pm} \longmapsto -\Gamma^{0,\pm}, \quad \Gamma^{i,\pm} \longmapsto \widetilde{\Gamma}^{i,\mp}, \quad \widetilde{\Gamma}^{i,\pm} \longmapsto -\Gamma^{i,\mp}. \tag{3.37}$$

From equation (3.28) we see that the coherent state $|\,C_6; p, \eta\rangle^{(0)}_{\mathrm{RR}}$ is invariant under the orientifolding by $\mathcal{G}$. Finally, using the same measure used in equation (3.21) for constructing position the eigenstate for NS-NS crosscap, we obtain the R-R part of the crosscap state

$$|\,C_6\rangle_{\mathrm{RR}} = \int \prod_{i=1}^{3} dp^i d\overline{p}^i \delta(p^i + e^{2\pi i k v_i}\overline{p}^i)\, \delta(z_0^i - e^{2\pi i k v_i}\overline{z}_0^i)\, |\,C_6; p\rangle_{\mathrm{RR}} \tag{3.38}$$

Finally, collecting the contributions from both the NS-NS and the R-R sectors in equation . (3.21) and (3.38) respectively, the crosscap state invariant under GSO projection and orientifolding is

$$|\,C_6\,\rangle = \frac{1}{\sqrt{2}}\Big[\,|\,C_6\,\rangle_{\mathrm{NSNS}} + |\,C_6\,\rangle_{\mathrm{RR}}\Big]. \tag{3.39}$$

This crosscap represents a canonical $O6$-plane *i.e.* which carries negative $D6$-brane charge. The sign is fixed by choosing $\mathcal{N}_C^{\mathrm{RR}}$ in (3.36) properly.

4. D-brane boundary state

Let us now proceed to discuss the construction of boundary states of the D0-branes and the D-instanton in the present model. The boundary states for the D0-branes are obtained, again, by solving an appropriate set of boundary conditions obtained in the world-sheet theory. The boundary state for the D-instanton is obtained from these by analytic continuation to an Euclidean time.

4.1 D0-brane

The boundary states of D0-branes has been worked out in ref. [3] in great detail. We briefly review their construction here. Let us begin with the boundary state for the D0-branes. The boundary conditions satisfied by the D0-branes are

$$\partial_\tau X^0(\sigma)\,|\,B0\,\rangle = 0, \quad \partial_\sigma X^{\pm}(\sigma)\,|\,B0\,\rangle = 0, \quad \partial_\sigma X^m(\sigma)\,|\,B0\,\rangle = 0, \tag{4.1}$$

$$(\psi^0 + i\eta\widetilde{\psi}^0)(\sigma)\,|\,B0\,\rangle = 0, \quad (\psi^{\pm} - i\eta\widetilde{\psi}^{\pm})(\sigma)\,|\,B0\,\rangle = 0, \quad (\psi^m - i\eta\widetilde{\psi}^m)(\sigma)\,|\,B0\,\rangle = 0, \tag{4.2}$$

for the space-time components of the bosonic and fermionic fields in the external four dimensions and

$$\partial_\sigma Z^i(\sigma)\,|\,B0\,\rangle = 0, \quad \partial_\sigma \overline{Z^i}(\tau = 0, \sigma)\,|\,B0\,\rangle = 0, \tag{4.3}$$

$$(\Psi^i - i\eta\widetilde{\Psi}^i)(\sigma)\,|\,B0\,\rangle = 0, \quad (\overline{\Psi^i} - i\eta\overline{\widetilde{\Psi}^i}(\sigma))\,|\,B0\,\rangle = 0, \tag{4.4}$$

for the internal components, where $m = 0, 3$, $i = 1, 2, 3$ and we have suppressed the temporal coordinate τ of the world-sheet from the notation. All the equations are at $\tau = 0$. These equations are valid both for the NS-NS and the R-R sector. As in the last section it is convenient to solve these equations in terms of the oscillators and momenta. The coherent states in the NS-NS and the R-R sectors are similar, except for the grading of the fermionic oscillators in the internal directions. In the NS-NS sector the index r of these oscillators, Ψ^i_r, are half-integral, taking values in $\mathbb{Z} + 1/2$, while in the R-R sector they are integral, taking values in $\mathbb{Z}$. However, formulas in the untwisted and the twisted sectors are rather different. So let us consider them in turn.

Untwisted sector

In terms of oscillators the equations for the untwisted NS-NS and R-R sectors become

$$P^0 \,|\, B0 \rangle = 0, \tag{4.5}$$

$$(\alpha^0_l + \widetilde{\alpha}^0_{-l}) \,|\, B0 \rangle = 0, \quad (\alpha^\pm_l - \widetilde{\alpha}^\pm_{-l}) \,|\, B0 \rangle = 0, \quad (\alpha^m_l - \widetilde{\alpha}^m_{-l}) \,|\, B0 \rangle = 0, \tag{4.6}$$

$$(\psi^0_r + i\eta\widetilde{\psi}^0_{-r}) \,|\, B0 \rangle = 0, \quad (\psi^\pm_r - i\eta\widetilde{\psi}^\pm_{-r}) \,|\, B0 \rangle = 0, \quad (\psi^m_r - i\eta\widetilde{\psi}^m_{-r}) \,|\, B0 \rangle = 0, \tag{4.7}$$

in the external four-dimensional part and

$$(\alpha^i_l - \widetilde{\alpha}^i_{-l}) \,|\, B0 \rangle = 0, \quad (\overline{\alpha^i_l} - \overline{\widetilde{\alpha}^i_{-l}}) \,|\, B0 \rangle = 0, \tag{4.8}$$

$$(\Psi^i_r - i\eta\widetilde{\Psi}^i_{-r}) \,|\, B0 \rangle = 0, \quad (\overline{\Psi^i_r} - i\eta\overline{\widetilde{\Psi}^i_{-r}}) \,|\, B0 \rangle = 0. \tag{4.9}$$

for the six-dimensional internal ones. From the equations (4.5)–(4.9) we observe that the momenta along all the Dirichlet directions, including the light-cone directions, P^3, $P^\pm$ and the momenta of the internal directions p^i, $\overline{p}^i$ as well as the spin structure η are the quantum numbers labelling the boundary states of the D0-brane and hence $|\, B_0 \rangle$ stands for $\big|\, P^\pm, P^3, p \big\rangle_{\substack{\text{NSNS}\\ \text{RR}}}, \eta$ in the untwisted sector.

A coherent state for the D0-brane is obtained by solving (4.5)–(4.9). The coherent states in the untwisted NS-NS and R-R sectors are built from the respective ground states. The ground state in the NS-NS sector is unique, carrying momenta $P^\pm$, P^3, p^i and $\overline{p}^i$ and is independent of the spin structure. Thus, the contribution from the untwisted NS-NS sector to the coherent state is

$$\begin{aligned}
\big|\, B0; P^\pm, P^3, p, \eta \big\rangle_{\text{NSNS,U}} = \exp\Bigg(&\sum_{l\in\mathbb{Z}} \frac{1}{l}(-\alpha^0_{-l}\widetilde{\alpha}^0_{-l} + \alpha^3_{-l}\widetilde{\alpha}^3_{-l}) + \sum_{\substack{i=1,2,3\\ l\in\mathbb{Z}}} \frac{1}{l}(\alpha^i_{-l}\overline{\widetilde{\alpha}^i_{-l}} + \overline{\alpha^i_{-l}}\widetilde{\alpha}^i_{-l}) \\
&+ i\eta \sum_{r\in\mathbb{Z}+\frac{1}{2}} (-\psi^0_{-r}\widetilde{\psi}^0_{-r} + \psi^3_{-r}\widetilde{\psi}^3_{-r}) + i\eta \sum_{\substack{i=1,2,3\\ r\in\mathbb{Z}+1/2}} (\Psi^i_{-r}\overline{\widetilde{\Psi}^i_{-r}} + \overline{\Psi^i_{-r}}\widetilde{\Psi}^i_{-r})\Bigg) \\
&\big|\, B0; P^\pm, P^3, p \big\rangle^{(0)}_{\text{NSNS,U}} ,
\end{aligned}$$

where $\big|\, B0; P^\pm, P^3, p \big\rangle^{(0)}_{\text{NSNS,U}}$ denotes the ground state in the untwisted NS-NS sector. The R-R ground states are degenerate, carrying momenta only along the extrenal light-cone directions and X^3, contributing

$$\big|\,B0; P^{\pm}, P^3, \eta\,\big\rangle_{\rm RR,U} = \exp\Bigg(\sum_{l\in\mathbb{Z}}\frac{1}{l}(-\alpha^0_{-l}\widetilde{\alpha}^0_{-l}+\alpha^3_{-l}\widetilde{\alpha}^3_{-l}) + \sum_{\substack{i=1,2,3\\ l\in\mathbb{Z}}}\frac{1}{l}(\alpha^i_{-l}\overline{\widetilde{\alpha}^i_{-l}}+\overline{\alpha^i_{-l}}\widetilde{\alpha}^i_{-l})$$
$$+\,i\eta\sum_{r\in\mathbb{Z}}(-\psi^0_{-r}\widetilde{\psi}^0_{-r}+\psi^3_{-r}\widetilde{\psi}^3_{-r}) + i\eta\sum_{\substack{i=1,2,3\\ r\in\mathbb{Z}}}(\Psi^i_{-r}\overline{\widetilde{\Psi}^i_{-r}}+\overline{\Psi^i_{-r}}\widetilde{\Psi}^i_{-r})\Bigg)\big|\,B0; P^{\pm}, P^3, \eta\,\big\rangle^{(0)}_{\rm RR,U}\,, \tag{4.10}$$

to the coherent state, where $\big|\,B0; P^{\pm}, P^3, \eta\,\big\rangle^{(0)}_{\rm RR,U}$ denotes the ground state in the untwisted R-R sector, given by [3],

$$\big|\,B0; P^{\pm}, P^3, p, \eta\,\big\rangle^{(0)}_{\rm RR,U} = \exp\Big[i\eta(-\Gamma^{0,+}\widetilde{\Gamma}^{0,+}+\sum_i\Gamma^{i,+}\widetilde{\Gamma}^{i,-})\Big]\,|----\rangle_L\otimes|-+++\rangle_R\,. \tag{4.11}$$

We now go on to consider the GSO projection and the orientifolding on the boundary states as we have done for the crosscap state in the preceding section.

4.1.1 GSO projection

Using the GSO projection operator from (2.6), we obtain the GSO-invariant combination of the boundary state in the untwisted NS-NS sector as,

$$\begin{aligned}&\big|\,D0; P^{\pm}, P^3, p\,\big\rangle_{\rm NSNS,U} = \\ &\frac{1}{\sqrt{3}}\frac{\mathcal{N}_0^{\rm NSNS,U}}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}\Big[\,\big|\,B0; P^{\pm}, P^3, p, +\,\big\rangle_{\rm NSNS,U} - \big|\,B0; P^{\pm}, P^3, p, -\,\big\rangle_{\rm NSNS,U}\Big],\end{aligned} \tag{4.12}$$

while the GSO-invariant state in the untwisted R-R sector is found to be

$$\big|\,D0; P^{\pm}, P^3\,\big\rangle_{\rm RR,U} = \frac{1}{\sqrt{3}}\frac{\mathcal{N}_0^{\rm RR,U}}{\sqrt{2}}\cdot\frac{1}{\sqrt{2}}\Big[\,\big|\,B0; P^{\pm}, P^3, +\,\big\rangle_{\rm RR,U} + \,\big|\,B0; P^{\pm}, P^3, -\,\big\rangle_{\rm RR,U}\Big]. \tag{4.13}$$

Finally, the position eigenstates are obtained by integrating on the available momenta. Assuming these fractional branes to be localized at the origin of the internal space which is also the location of orbifold singularity, the postion eigenstates in the NS-NS and the R-R sectors are, respectively,

$$\big|\,D0\,\big\rangle_{\rm NSNS,U} = \int dP^+\;dP^-\;dP^3\prod_{i=1}^3\;dp^i d\overline{p}^i\,\big|\,D0; P^{\pm}, P^3, p\,\big\rangle_{\rm NSNS,U} \tag{4.14}$$

$$\big|\,D0\,\big\rangle_{\rm RR,U} = \int dP^+\;dP^-\;dP^3\,\big|\,D0; P^{\pm}, P^3\,\big\rangle_{\rm RR,U} \tag{4.15}$$

We have thus obtained the contributions to the boundary state of the D0-brane from the untwisted NS-NS and R-R sectors. The untwisted part of the D0-brane boundary state in the orbifold theory is

$$\big|\,D0\,\big\rangle^{\rm U}_{\rm orb} = \frac{1}{\sqrt{2}}\Big[\,\big|\,D0\,\big\rangle_{\rm NSNS,U} + \big|\,D0\,\big\rangle_{\rm RR,U}\Big] \tag{4.16}$$

Having obtained the GSO-invariant untwisted state for the D0-brane in the orbifold theory, let us now consider orientifolding them.

4.1.2 Orientifolding

From the action given in (2.6) for the states, we find that the NS-NS part remains invariant under the orientifolding. In the R-R sector, the exponential factor remains unaltered, while the ground state (4.13) flips sign. Hence the orientifold invariant untwisted part is obtained by taking a linear sum of brane and anti-brane boundary states. So the untwisted part of the D0-brane boundary state in the orientifold theory looks like

$$|\,D0\,\rangle^{\rm U}_{\rm orientifold} = \sqrt{2}\ |\,D0\,\rangle_{\rm NSNS,U} \tag{4.17}$$

Hence it looks a like a non-BPS fractional D0-brane.

Twisted sectors

Let us now consider the contributions from the twisted sectors. By substituting the expansions (A.6) and (A.13) in (4.1) — (4.4) we obtain the boundary conditions to be satisfied by the k-th twisted sector in terms of the oscillators as,

$$P^0\,|\,B0;k\,\rangle = 0 \tag{4.18}$$

$$(\alpha_l^{\pm} - \widetilde{\alpha}^{\pm}_{-l})\,|\,B0;k\,\rangle = 0, \quad (\alpha_l^{0} + \widetilde{\alpha}^{0}_{-l})\,|\,B0;k\,\rangle = 0, \quad (\alpha_l^{\mu} - \widetilde{\alpha}^{\mu}_{-l})\,|\,B0;k\,\rangle = 0, \tag{4.19}$$

$$(\alpha^i_{l+kv_i} - \widetilde{\alpha}^i_{-l-kv_i})\,|\,B0;k\,\rangle = 0, \quad (\overline{\alpha}^i_{l-kv_i} - \widetilde{\overline{\alpha}}^i_{-l+kv_i})\,|\,B0;k\,\rangle = 0, \tag{4.20}$$

for the bosonic oscillators and

$$(\psi_r^{\pm} - i\eta\widetilde{\psi}^{\pm}_{-r})\,|\,B0;k\,\rangle = 0, \quad (\psi_r^{0} + i\eta\widetilde{\psi}^{0}_{-r})\,|\,B0;k\,\rangle = 0, \quad (\psi_r^{3} - i\eta\widetilde{\psi}^{3}_{-r})\,|\,B0;k\,\rangle = 0, \tag{4.21}$$

$$(\Psi^i_{r+kv_i} - i\eta\widetilde{\Psi}^i_{-r-kv_i})\,|\,B0;k\,\rangle = 0, \quad (\overline{\Psi}^i_{r-kv_i} - i\eta\widetilde{\overline{\Psi}}^i_{-r+kv_i})\,|\,B0;k\,\rangle = 0, \tag{4.22}$$

for the fermionic oscillators, where $|\,B0;k\,\rangle$ represents the boundary state in the k-th twisted sector. In the R-R sector r is an integer in equations (4.21) and (4.22), while it is half-integral in the NS-NS sector.

A coherent state for $|\,B0;k\,\rangle$ is constructed from the twisted sector ground states. The twisted NS-NS and R-R ground states are the same as the ones used in constructing the crosscap state, with degenerate twisted R-R ground states. The contribution of the twisted sector $|\,B0;k\,\rangle$ to the boundary state is

$$\begin{aligned}
&\big|\,B0;P^{\pm},P^3,\eta;k\,\big\rangle_{\rm NSNS,T} = \\
&\exp\Bigg[\sum_{l\in\mathbb{Z}}\frac{1}{l}(-\alpha^0_{-l}\widetilde{\alpha}^0_{-l} + \alpha^3_{-l}\widetilde{\alpha}^3_{-l}) + i\eta\sum_{r\in\mathbb{Z}+1/2}(-\psi^0_{-r}\widetilde{\psi}^0_{-r} + \psi^3_{-r}\widetilde{\psi}^3_{-r}) \\
&\qquad + \sum_{\substack{i=1,2,3\\ l\in\mathbb{Z}}}\left(\frac{1}{l-kv_i}\alpha^i_{-l+kv_i}\widetilde{\overline{\alpha}}^i_{-l+kv_i} + \frac{1}{l+kv_i}\overline{\alpha}^i_{-l-kv_i}\widetilde{\alpha}^i_{-l-kv_i}\right) \\
&\qquad + i\eta\sum_{\substack{i=1,2,3\\ r\in\mathbb{Z}+1/2}}\left(\Psi^i_{-r+kv_i}\widetilde{\overline{\Psi}}^i_{-r+kv_i} + \overline{\Psi}^i_{-r-kv_i}\widetilde{\Psi}^i_{-r-kv_i}\right)\Bigg]\,\big|\,B0;P^{\pm},P^3;k\,\big\rangle^{(0)}_{\rm NSNS,T}\,,
\end{aligned} \tag{4.23}$$

from the NS-NS sector and

$$
\begin{aligned}
\left| B0; P^{\pm}, P^3, \eta; k \right\rangle_{\mathrm{RR,T}} = \exp\Bigg[& \sum_{l\in\mathbb{Z}} \frac{1}{l}(-\alpha^0_{-l}\widetilde{\alpha}^0_{-l} + \alpha^3_{-l}\widetilde{\alpha}^3_{-l}) + i\eta \sum_{r\in\mathbb{Z}} (-\psi^0_{-r}\widetilde{\psi}^0_{-r} + \psi^3_{-r}\widetilde{\psi}^3_{-r}) \\
& + \sum_{\substack{i=1,2,3 \\ l\in\mathbb{Z}}} \left(\frac{1}{l-kv_i}\alpha^i_{-l+kv_i}\widetilde{\overline{\alpha}}^i_{-l+kv_i} + \frac{1}{l+kv_i}\overline{\alpha}^i_{-l-kv_i}\widetilde{\alpha}^i_{-l-kv_i} \right) \\
& + i\eta \sum_{\substack{i=1,2,3 \\ r\in\mathbb{Z}}} \left(\Psi^i_{-r+kv_i}\widetilde{\overline{\Psi}}^i_{-r+kv_i} + \overline{\Psi}^i_{-r-kv_i}\widetilde{\Psi}^i_{-r-kv_i} \right) \Bigg] \left| B0; P^{\pm}, P^3, \eta; k \right\rangle^{(0)}_{\mathrm{RR,T}},
\end{aligned} \tag{4.24}
$$

from the R-R sector. The ground states are

$$
\left| B0; P^{\pm}, P^3; k=\pm1 \right\rangle^{(0)}_{\mathrm{NSNS,T}} = \prod_i \sigma^i_{\pm}\widetilde{\sigma}^i_{\pm} \left| 0, \pm\frac{1}{3}, \pm\frac{1}{3}, \pm\frac{1}{3}, \right\rangle_L \otimes \left| 0, \pm\frac{1}{3}, \pm\frac{1}{3}, \pm\frac{1}{3}, \right\rangle_R \tag{4.25}
$$

$$
\left| B0; P^{\pm}, P^3; k=\pm1 \right\rangle^{(0)}_{\mathrm{RR,T}} = \prod_i \sigma^i_{k,\pm}\widetilde{\sigma}^i_{k,\pm} \left| \mp\frac{1}{2}, \pm\frac{1}{6}, \pm\frac{1}{6}, \pm\frac{1}{6}, \right\rangle_L \otimes \left| \mp\frac{1}{2}, \pm\frac{1}{6}, \pm\frac{1}{6}, \pm\frac{1}{6}, \right\rangle_R \tag{4.26}
$$

4.1.3 GSO projection

The GSO-invariant states are given by

$$
\begin{aligned}
& \left| D0; P^{\pm}, P^3; k \right\rangle_{\substack{\mathrm{NSNS,T} \\ \mathrm{RR,T}}} = \qquad (4.27) \\
& \frac{1}{\sqrt{3}} \frac{\mathcal{N}^{\mathrm{T}}_{\substack{\mathrm{NSNS,T} \\ \mathrm{RR,T}}}}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \Big[\left| B0; P^{\pm}, P^3, +; k \right\rangle_{\substack{\mathrm{NSNS,T} \\ \mathrm{RR,T}}} \mp \left| B0; P^{\pm}, P^3, -; k \right\rangle_{\substack{\mathrm{NSNS,T} \\ \mathrm{RR,T}}} \Big],
\end{aligned}
$$

where the upper and lower signs are for NS-NS and R-R sectors respectively.

The contribution to the boundary state of the D0-brane from the k-th twisted sector is given by the position eigenstate obtained by integrating over the transverse momenta as,

$$
\left| D0; k \right\rangle_{\substack{\mathrm{NSNS,T} \\ \mathrm{RR,T}}} = \int dP^+ \, dP^- \, dP^3 \, \left| D0; P^{\pm}, P^3; k \right\rangle_{\substack{\mathrm{NSNS,T} \\ \mathrm{RR,T}}} \tag{4.28}
$$

So the twisted part of the D0-brane in the orbifold theory is

$$
\left| D0 \right\rangle^{\mathrm{T}}_{\mathrm{orb}} = \frac{1}{\sqrt{2}} \sum_{k=\pm1} \Big[\epsilon^k_1 \epsilon^k_2 \left| D0; k \right\rangle_{\mathrm{NSNS,T}} + \epsilon^k_1 \left| D0; k \right\rangle_{\mathrm{RR,T}} \Big], \tag{4.29}
$$

where $\epsilon^k_i = \pm1$ for $i = 1, 2$ and $k = \pm1$, ϵ^k_1 denotes the twisted R-R charges of these branes under twisted R-R field from k-th sector. Finally, $\epsilon^k_1\epsilon^k_2$ denote the phase of the untwisted NS-NS sectors [57, 61].

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4.1.4 Orientifolding

The orientifolding operation keeps the part inside the exponential in equation . (4.23) and (4.24) invariant. Moreover, the twisted NS-NS ground state turns out to be invariant in both $k = \pm 1$ sectors. However, as in the untwisted sector, the twisted R-R sector ground states picks up a minus sign. So the orientifold invariant twisted part of the D0-brane is the sum of brane and anti-bane system.

$$|\, D0\,\rangle^{\rm T}_{\rm orientifold} = \sqrt{2} \sum_{k=\pm 1} \epsilon_1^k \epsilon_2^k \,|\, D0; k\,\rangle_{\rm NSNS,T} \tag{4.30}$$

Therefore, the actual boundary state of the D0-brane surviving orientifold projection do not have any R-R part. It is given by the sum of (4.17) and (4.30),

$$\begin{aligned} |\, D0\,\rangle_{\rm orientifold} &= |\, D0\,\rangle^{\rm U}_{\rm orientifold} + |\, D0\,\rangle^{\rm T}_{\rm orientifold} \\ &= \sqrt{2}\left[|\, D0\,\rangle_{\rm NSNS,U} + \sum_{k=\pm 1} \epsilon_1^k \epsilon_2^k \,|\, D0; k\,\rangle_{\rm NSNS,T} \right] \end{aligned} \tag{4.31}$$

This is consistent with the fact that there is no one-form R-R field in the closed string spectrum.

4.2 D-instanton

In Type-IIA theory there is no BPS D-instanton state. We can always write down the boundary state of a non-BPS D-instanton, which consists only of the NS-NS part. It can be obtained by flipping the sign in front of the timelike oscillators in the expression of D0 boundary state. The untwisted part is

$$\begin{aligned} |\, D(-1), P, \eta\,\rangle_{\rm NSNS,U} = \exp\Bigg[& \sum_{\substack{l\in\mathbb{Z} \\ \mu=0,3}} \frac{1}{l}\, \alpha^\mu_{-l} \widetilde{\alpha}^\mu_{-l} + i\eta \sum_{\substack{r\in\mathbb{Z}+\frac{1}{2} \\ \mu=0,3}} (-)^r \psi^\mu_{-r} \widetilde{\psi}^\mu_{-r} \\ & + \sum_{\substack{i=1,2,3 \\ l\in\mathbb{Z}_+}} \frac{1}{l} \left(\alpha^i_{-l} \overline{\widetilde{\alpha}^i}_{-l} + \overline{\alpha^i}_{-l} \widetilde{\alpha}^i_{-l} \right) + i\eta \sum_{\substack{i=1,2,3 \\ r\in\mathbb{Z}+\frac{1}{2}}} \left(\Psi^i_{-r} \overline{\widetilde{\Psi}^i}_{-r} + \overline{\Psi^i}_{-r} \widetilde{\Psi}^i_{-r} \right) \Bigg] |\, B_{-1}, P\,\rangle^{(0)}_{\rm NSNS}\,. \end{aligned} \tag{4.32}$$

Here, the metric along μ directions is Euclidean, $\delta^{\mu\nu}$, $\mu, \nu = 0, 3$. Generically, in the orbifold theory such a D-instanton will be sourced by twisted sector fields, so it also has a twisted NS-NS part

$$\begin{aligned} \big|\, D(-1), P^\pm, P^\mu, \eta\,\big\rangle_{\rm NSNS,T} = \exp\Bigg[& \sum_{l\in\mathbb{Z}} \sum_{\mu=0,3} \frac{1}{l} \alpha^\mu_{-l} \widetilde{\alpha}^\mu_{-l} + i\eta \sum_{r\in\mathbb{Z}+1/2} \sum_{\mu=0,3} \psi^\mu_{-r} \widetilde{\psi}^\mu_{-r}) \\ & + \sum_{\substack{i=1,2,3 \\ l\in\mathbb{Z}}} \left(\frac{1}{l - kv_i} \alpha^i_{-l+kv_i} \overline{\widetilde{\alpha}}^i_{-l+kv_i} + \frac{1}{l + kv_i} \overline{\alpha}^i_{-l-kv_i} \widetilde{\alpha}^i_{-l-kv_i} \right) \\ & + i\eta \sum_{\substack{i=1,2,3 \\ r\in\mathbb{Z}+1/2}} \left(\Psi^i_{-r+kv_i} \overline{\widetilde{\Psi}}^i_{-r+kv_i} + \overline{\Psi}^i_{-r-kv_i} \widetilde{\Psi}^i_{-r-kv_i} \right) \Bigg] \big|\, P^\pm, P^\mu\,\big\rangle^{(0)}_{\rm NSNS,T}\,, \end{aligned} \tag{4.33}$$

The GSO-invariant boundary state for the D(−1)-brane is

$$|\,D(-1)\,\rangle = \frac{1}{\sqrt{2}}\left[|\,D(-1)\,\rangle_{\rm NSNS,U} + \sum_{k=\pm 1}\epsilon_1^k\epsilon_2^k\,|\,D(-1)\,\rangle_{\rm NSNS,T}\right], \tag{4.34}$$

where $\epsilon_i^k = \pm 1$ for all $k - \pm 1$ and $i = 1, 2$, as before and

$$|\,D(-1)\,\rangle_{\rm NSNS,U} = \frac{1}{\sqrt{3}}\frac{\mathcal{N}_{-1}^{\rm U}}{\sqrt{2}}\int\prod_{M=0}^{9} dP^M\ \frac{1}{\sqrt{2}}\Big[\,|\,D(-1); P, +\,\rangle_{\rm NSNS} - |\,D(-1); P, -\,\rangle_{\rm NSNS}\,\Big] \tag{4.35}$$

and

$$\begin{aligned}|\,D(-1)\,\rangle_{\rm NSNS,T} = \frac{1}{\sqrt{3}}\frac{\mathcal{N}^{\rm T}}{\sqrt{2}}\int dP^+dP^-dP^0dP^3\ \frac{1}{\sqrt{2}}\Big[\,\big|\,D(-1), P^{\pm}, P^{\mu}; +\,\big\rangle_{\rm NSNS,T}\\ -\,\big|\,D(-1), P^{\pm}, P^{\mu}; -\,\big\rangle_{\rm NSNS,T}\Big]\end{aligned} \tag{4.36}$$

Here $\mathcal{N}_{-1}$ is the normalization factor to be determined in a self-consistent way later[6]. Obviously, this is not a *stable* D-instanton as it contains tachyon in its spectrum.

We shall be interested in a D-instanton whose boundary state does not have a coupling to twisted NS-NS state [62, 63]. This can be obtained as a linear combination of two D-instanton boundary states given in equation (4.34), with opposite coupling to twisted NS-NS fields for each k.

$$|\,\widetilde{D(-1)}\,\rangle = |\,D(-1)\,\rangle_{\rm NSNS,U} \tag{4.37}$$

5. One loop open string amplitudes

Let us now proceed to compute the annulus and Möbius amplitudes. The D0-brane boundary state, as given in equation (4.31), is obviously non-BPS and hence contains tachyon in its spectrum. Moreover, since this boundary state is a sum of D0-$\overline{\rm D0}$-pair, this tachyon is a complex field. Thequestion is whether the orientifold can get rid of it and make it stable non-BPS D0-brane. We don't expect that the orientifold projection can get rid of two *real* tachyons, otherwise it would be a novel feature of having a theory with both BPS D0-brane(bulk or non-fractional) and stable non-BPS fractional D0-branes. Unfortunately, we could not prove it by a direct computation in open string language. However, boundary state analysis of this section clearly indicates that the tachyon is *not* projected out in the orientifold theory.

The absence of an R-R part in the boundary state of the D0-branes simplifies the considerations as it now suffices to consider the NS-NS amplitudes only. Furthermore, the crosscap state does not have a twisted piece. Hence, for twisted sectors the Möbius amplitude vanishes.

[6]Since Type IIA D-instanton can be obtained from a D-instanton-anti-D-instanton pair in Type IIB and modding it out by $(-1)^{F_L}$, we know that $\mathcal{N}_{-1}$ is $\sqrt{2}$-times bigger than the BPS D-instanton. We shall get the same fact by demanding that it becomes stable in the orientifold theory.

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For D-instanton in (4.37), however, the open string amplitudes, annulus plus Möbius, turns out to be free of tachyons.

Throughout this and following sections, t and ℓ will denote the tree (closed) and open (loop) channel Euclidean time respectively. Similarly, $\widetilde{q} = e^{-2\pi t}$ and $q = e^{-\pi\ell}$ will denote the tree (closed) and open (loop) channel modular parameters. The parameters t and l for various geometries are related as follows [50]

$$t = \begin{cases} 1/2\ell & \text{for Annulus,} \\ 1/8\ell & \text{for Möbius,} \\ 1/4\ell & \text{for Klein bottle.} \end{cases} \tag{5.1}$$

5.1 D0-brane

We first compute the annulus and the Möbius amplitudes associated with the D0-brane. As the results are different for untwisted and twisted sectors we discuss them separately. The full D0-brane open string amplitudes is a sum of annulus and Möbius in both untwisted and twisted sectors.

Untwisted sector

5.1.1 Annulus amplitude

As the D0-brane boundary states occur as in (4.17) and (4.12) we need the following amplitudes

$$\begin{aligned}
\int_0^\infty dt \;{}_{\rm NSNS,U}\langle D0;+|\,e^{-tH_c}\,|D0;+\rangle_{\rm NSNS,U} &= \int_0^\infty dt \;{}_{\rm NSNS,U}\langle D0;-|\,e^{-tH_c}\,|D0;-\rangle_{\rm NSNS,U} \\
&= \frac{1}{3}\cdot\frac{1}{2}\cdot\frac{4\cdot(\mathcal{N}_0^{\rm NSNS,U})^2}{2}\int_0^\infty \frac{d\ell}{2\ell^{\frac{3}{2}}}\frac{f_3(q)^8}{f_1(q)^8} \\
\int_0^\infty dt \;{}_{\rm NSNS,U}\langle D0;+|\,e^{-tH_c}\,|D0;-\rangle_{\rm NSNS,U} &= \int_0^\infty dt \;{}_{\rm NSNS,U}\langle D0;-|\,e^{-tH_c}\,|D0;+\rangle_{\rm NSNS,U} \\
&= \frac{1}{3}\cdot\frac{1}{2}\cdot\frac{4\cdot(\mathcal{N}_0^{\rm NSNS,U})^2}{2}\int_0^\infty \frac{d\ell}{2\ell^{\frac{3}{2}}}\frac{f_2(q)^8}{f_1(q)^8}.
\end{aligned} \tag{5.2}$$

Using (5.2) we write the untwisted annulus amplitude of D0-brane in the orientifold

$$\begin{aligned}
\mathcal{A}_{0-0}^{\rm orientifold} &= \int_0^\infty dt \;{}_{\rm orientifold}\langle D0|\,e^{-tH_c}\,|D0\rangle_{\rm orientifold} \\
&= \frac{1}{3}\cdot\frac{1}{2}\cdot\frac{4\cdot(\mathcal{N}_0^{\rm NSNS,U})^2}{2}\int_0^\infty \frac{d\ell}{2\ell^{\frac{3}{2}}}\left(\frac{f_3(q)^8}{f_1(q)^8} - \frac{f_2(q)^8}{f_1(q)^8}\right).
\end{aligned} \tag{5.3}$$

Since the numerical factors appearing in the above formulas are of different origins, we have shown them explicitly, for example, the first factor of $\frac{1}{3}$ is to reproduce the orbifold projector correctly, the second $\frac{1}{2}$ factor is due to the GSO and the third one is for the orientifolding. Comparing this expression with the orbifold case as given in [3] we can see that orientifolding takes away the contribution from R-R sector and thus it has a tachyonic contribution which will otherwise be absent.

5.1.2 Möbius amplitude

The Möbius amplitude is obtained from two amplitudes, namely, the amplitude between the crosscap and the D0-brane states with both having positive spin-structure,

$$\begin{aligned}
&\int_0^\infty dt \;{}_{\rm NSNS}\langle C_6;+|\,\Omega\mathcal{R}(-1)^{F_L}e^{-tH_c}\,|D0;+\rangle_{\rm NSNS,U}\\
&\qquad\qquad = \int_0^\infty dt \;{}_{\rm NSNS}\langle C_6;-|\,\Omega\mathcal{R}(-1)^{F_L}e^{-tH_c}\,|D0;-\rangle_{\rm NSNS,U}\\
&\qquad\qquad = \frac{1}{3}\cdot\frac{1}{2}\cdot\frac{\mathcal{N}_c^{\rm NSNS}\cdot\mathcal{N}_0^{\rm NSNS,U}}{2}\cdot 2\int_0^\infty \frac{d\ell}{2\ell^{\frac{3}{2}}}\,\frac{f_3(q^2)^4}{f_1(q^2)^4},
\end{aligned} \tag{5.4}$$

and the amplitude in which the crosscap and the D0-brane have opposite spin-structures,

$$\begin{aligned}
&\int_0^\infty dt \;{}_{\rm NSNS}\langle C_6;+|\,\Omega\mathcal{R}(-1)^{F_L}e^{-tH_c}\,|D0;-\rangle_{\rm NSNS,U}\\
&\qquad\qquad = {}_{\rm NSNS}\langle C_6;-|\,\Omega\mathcal{R}(-1)^{F_L}e^{-tH_c}\,|D0;+\rangle_{\rm NSNS,U} \qquad (5.5)\\
&\qquad\qquad = \frac{1}{3}\cdot\frac{1}{2}\cdot\frac{\mathcal{N}_c^{\rm NSNS}\cdot\mathcal{N}_0^{\rm NSNS,U}}{2}\cdot 2\int_0^\infty \frac{d\ell}{2\ell^{\frac{3}{2}}}\,\frac{f_3(q^2)^4}{f_1(q^2)^4}.
\end{aligned}$$

Since the D0-brane in the orientifold does not have an R-R part, the total untwisted D0-brane Möbius amplitude

$$\mathcal{M}_{0-0} = \int_0^\infty dt \;{}_{\rm NSNS}\langle C_6|\,e^{-tH_c}\,|D0\rangle_{\rm NSNS,U} = \frac{1}{3}\cdot\frac{1}{2}\cdot\frac{\mathcal{N}_c\cdot\mathcal{N}_0}{2}\cdot 2\int_0^\infty \frac{d\ell}{2\ell^{\frac{3}{2}}}\Big[\frac{f_3(q^2)^4}{f_1(q^2)^4} - \frac{f_3(q^2)^4}{f_1(q^2)^4}\Big] \tag{5.6}$$

vanishes identically. Therefore, the tachyonic degree of freedom survives and makes the D0-brane unstable in the untwisted sector.

Twisted sector

Since there is no contribution to the Möbius amplitude from the twisted sectors, as the order of the orbifolding group is odd in the present case, the complete amplitude is given by the annulus. Moreover, the entire contribution comes from the twisted NS-NS sector. Using the expressions

$$\begin{aligned}
\int_0^\infty dt\ {}_{\rm NSNS,T}\langle D0;+,k|e^{-tH_c}|D0;+,k\rangle_{\rm NSNS,T} \\
&= \int_0^\infty dt\ {}_{\rm NSNS,T}\langle D0;-,k|\,e^{-tH_c}\,|D0;-,k\rangle_{\rm NSNS,T} \\
&= \frac{1}{3}\cdot\frac{1}{2}\cdot\frac{4\cdot(\mathcal{N}_0^{\rm NSNS,T})^2}{2}\int_0^\infty \frac{d\ell}{2\ell^{\frac{3}{2}}}\ \vartheta_3(0|i\ell)\prod_{i=1}^3 \sin(\pi k\nu_i)\frac{\vartheta_3(k\nu_i\ell|i\ell)}{\vartheta_1(k\nu_i\ell|i\ell)} \\
\int_0^\infty dt\ {}_{\rm NSNS,T}\langle D0;+,k|e^{-tH_c}|D0;-,k\rangle_{\rm NSNS,T} \\
&= \int_0^\infty dt\ {}_{\rm NSNS,T}\langle D0;-,k|\,e^{-tH_c}\,|D0;+,k\rangle_{\rm NSNS,T} \\
&= \frac{1}{3}\cdot\frac{1}{2}\cdot\frac{4\cdot(\mathcal{N}_0^{\rm NSNS,T})^2}{2}\int_0^\infty \frac{d\ell}{2\ell^{\frac{3}{2}}}\ \vartheta_2(0|i\ell)\prod_{i=1}^3 \sin(\pi k\nu_i)\frac{\vartheta_2(k\nu_i\ell|i\ell)}{\vartheta_1(k\nu_i\ell|i\ell)}.
\end{aligned} \tag{5.7}$$

where $k = \pm 1$ correspond to the twisted sectors, we get the annulus amplitude

$$\begin{aligned}
\int_0^\infty dt\ {}_{\rm NSNS,T}\langle D0;T;k|\,e^{-tH_c}\,|D0;T;k\rangle_{\rm NSNS,T} \\
= \frac{1}{3}\cdot\frac{1}{2}\cdot\frac{4\cdot(\mathcal{N}_0^{\rm NSNS,T})^2}{2}\int_0^\infty \frac{d\ell}{2\ell^{\frac{3}{2}}}\sum_{k=\pm1}\Bigg(&\vartheta_3(0|i\ell)\prod_{i=1}^3 \sin(\pi k\nu_i)\frac{\vartheta_3(k\nu_i\ell|i\ell)}{\vartheta_1(k\nu_i\ell|i\ell)} \\
&-\vartheta_2(0|i\ell)\prod_{i=1}^3 \sin(\pi k\nu_i)\frac{\vartheta_2(k\nu_i\ell|i\ell)}{\vartheta_1(k\nu_i\ell|i\ell)}\Bigg),
\end{aligned} \tag{5.8}$$

from the twisted sector. Expanding in q we find that there are tachyonic contribution which makes this boundary state unstable.

5.2 D-instanton

5.2.1 Annulus amplitude

As in D0-brane case, using standard procedure, we can work out the annulus amplitude for D-instanton, using (4.35) and (4.37) as the boundary state.

$$\mathcal{A}_{(-1)-(-1)} = \int_0^\infty dt\langle\widetilde{D(-1)}|e^{-tH_c}|\widetilde{D(-1)}\rangle = \frac{1}{3}\int_0^\infty \frac{d\ell}{2\ell}\,\frac{(\mathcal{N}_{-1}^{\rm NSNS,U})^2}{2}\,\frac{\left[f_3^8(q)-f_2^8(q)\right]}{f_1^8(q)} \tag{5.9}$$

5.2.2 Möbius amplitude

In order to calculate the Möbius amplitude associated with the D-instanton we need the following expressions,

$$\int_0^\infty dt \ {}_{\rm NSNS}\langle C_6;\pm|\,e^{-tH_c}\,|D(-1),\pm\rangle_{\rm NSNS,U} = 2\ i^{1/4}\int_0^\infty \frac{d\ell}{(2\ell)^{3/2}}\ e^{i\pi/4}\frac{f_4^2(iq)f_3^3(q^2)}{f_2^2(iq)f_1^3(q^2)},$$
$$\int_0^\infty dt \ {}_{\rm NSNS}\langle C_6;\pm|\,e^{-tH_c}\,|D(-1),\mp\rangle_{\rm NSNS,U} = 2\ i^{1/4}\int_0^\infty \frac{d\ell}{(2\ell)^{3/2}}\ e^{-i\pi/4}\frac{f_3^2(iq)f_3^3(q^2)}{f_2^2(iq)f_1^3(q^2)}. \tag{5.10}$$

The total Möbius amplitude in this case is

$$\mathcal{M}_{6-(-1)}+\mathcal{M}^*_{6-(-1)} = \frac{1}{3}\int_0^\infty \frac{d\ell}{2\ell}\ \frac{1}{2^2}\frac{\mathcal{N}_C^{\rm NSNS}\mathcal{N}_{-1}^{\rm NSNS,U}}{2\sqrt{2}}\ \frac{2^5\cdot 2}{\sqrt{2}}i^{1/4}$$
$$\left[\frac{1}{\sqrt{2}}\Big(\frac{f_4^2(i\widetilde{q})f_3^3(\widetilde{q}^2)}{f_2^2(i\widetilde{q})f_1^3(\widetilde{q}^2)}-\frac{f_3^2(i\widetilde{q})f_3^3(\widetilde{q}^2)}{f_2^2(i\widetilde{q})f_1^3(\widetilde{q}^2)}\Big)\right. \tag{5.11}$$
$$\left.+\frac{i}{\sqrt{2}}\Big(\frac{f_4^2(i\widetilde{q})f_3^3(\widetilde{q}^2)}{f_2^2(i\widetilde{q})f_1^3(\widetilde{q}^2)}+\frac{f_3^2(i\widetilde{q})f_3^3(\widetilde{q}^2)}{f_2^2(i\widetilde{q})f_1^3(\widetilde{q}^2)}\Big)\right]$$

5.3 Analysis of D-instanton partition function for the tachyon

Since the D(-1)-brane is a non-BPS object, the open string spectra on it has no GSO projection, *i.e.*

$$\mathcal{A}_{(-1)-(-1)}+\mathcal{M}_{6-(-1)}+\mathcal{M}^*_{6-(-1)} = \frac{1}{3}\cdot\frac{1}{2}\int\frac{dl}{2l}\Bigg(\mathrm{Tr}_{\rm NS}\Big[(1+\Omega\mathcal{R}(-1)^{F_L})q^{H_o}\Big]$$
$$-\Big[\mathrm{Tr}_{\rm R}(1+\Omega\mathcal{R}(-1)^{F_L})q^{H_o}\Big]\Bigg). \tag{5.12}$$

We can write this expression as a sum of two terms with opposite GSO projectors [68]. Thus

$$\mathcal{A}_{(-1)-(-1)}+\mathcal{M}_{6-(-1)}+\mathcal{M}^*_{6-(-1)} = \frac{1}{3}\cdot\frac{1}{2}\int\frac{dl}{2l}(Z_{\rm NS+}(q)+Z_{\rm NS-}(q)+Z_{\rm R}(q)), \tag{5.13}$$

where

$$Z_{\rm NS\pm}(q) = \mathrm{Tr}_{\rm NS}\Big[\frac{(1+\Omega\mathcal{R}(-1)^{F_L})}{2}(\frac{1\pm(-1)^F}{2})q^{H_o}\Big]. \tag{5.14}$$

Since the tachyon is odd under $(-1)^F$, it lives in the sector with partition function $Z_{\rm NS-}(q)$. Similarly, we can know about the massless scalars, if we analyze $Z_{\rm NS+}(q)$, as the latter are $(-1)^F$ even. Let us now assume that[7]

$$\mathcal{N}_c^{\rm NSNS} = ix. \tag{5.15}$$

[7] The factor i in the normalization $\mathcal{N}_C^{\rm NSNS}$ should not be conjugated when considering the conjugate crosscap state. This is a BPZ conjugation of CFT, not the standard quantum mechanical hermitian conjugation. See [57] for a discussion on this issue.

From the annulus and Möbius amplitudes, given in equation (5.9) and (5.11), we now write down the expression for $Z_{\mathrm{NS}\pm}(q)$ and $Z_{\mathrm{R}}(q)$.

$$Z_{\mathrm{NS}-}(q) = \frac{(\mathcal{N}_{-1}^{\mathrm{NSNS,U}})^2}{2}\frac{[f_3^8(q)+f_4^8(q)]}{2f_1^8(q)} - i^{1/4}\cdot\frac{x\mathcal{N}_{-1}^{\mathrm{NSNS,U}}}{2\sqrt{2}}\left[\frac{f_4^2(iq)f_3^3(q^2)}{f_2^2(iq)f_1^3(q^2)}+\frac{f_3^2(iq)f_3^3(q^2)}{f_2^2(iq)f_1^3(q^2)}\right], \tag{5.16}$$

$$Z_{\mathrm{NS}+}(q) = \frac{(\mathcal{N}_{-1}^{\mathrm{NSNS,U}})^2}{2}\frac{[f_3^8(q)-f_4^8(q)]}{2f_1^8(q)} + i^{1/4}\cdot\frac{x\mathcal{N}_{-1}^{\mathrm{NSNS,U}}}{2\sqrt{2}}i\left[\frac{f_4^2(iq)f_3^3(q^2)}{f_2^2(iq)f_1^3(q^2)}-\frac{f_3^2(iq)f_3^3(q^2)}{f_2^2(iq)f_1^3(q^2)}\right], \tag{5.17}$$

$$Z_{\mathrm{R}}(q) = -\frac{(\mathcal{N}_{-1}^{\mathrm{NSNS,U}})^2}{2}\frac{f_2^8(q)}{f_1^8(q)}. \tag{5.18}$$

The normalization of the crosscap is known from the compact cousin of the model under consideration [39],

$$x = 2^{5/2}. \tag{5.19}$$

If we choose

$$\mathcal{N}_{-1}^{\mathrm{NSNS,U}} = 2^5, \tag{5.20}$$

making an expansion of these partition functions for small q, we find that the tachyon gets projected out in $Z_{\mathrm{NS}-}(q)$

$$Z_{\mathrm{NS}-}(q) \sim 6q + \mathcal{O}(q^2). \tag{5.21}$$

Similarly, we find

$$Z_{\mathrm{NS}+}(q) \sim 10 + \mathcal{O}(q). \tag{5.22}$$

This reflects the fact that on this D-instanton world-volume we have ten massless modes corresponding to the freedom of translating it along its ten transverse dimensions.

6. K-theory & orientifold

Having thus obtained the crosscap state and the D-brane boundary states let us now discuss the K-theory associated with the orientifold model under consideration. The K-groups yield the different charges of the branes via the Chern characters. D0-branes on orbifolds can be identified as objects of the derived category of an Abelian category. The latter can be described either as the category of coherent sheaves on $\mathbb{P}^2$ or as the category of representations of the quiver associated with $\mathbb{P}^2$. These two definitions of the category are relevant in two different regimes of the Kähler moduli space of the $\mathbb{P}^2$. The coherent sheaves portray the D0-branes on $\mathbb{P}^2$ in the large volume region, while the representations of the quiver limn them in the orbifold limit, wherein the volume of the $\mathbb{P}^2$ shrinks [26, 3]. In either description, the different branes are given by the Grothendieck group K_0 of the Abelian category [66, 58]. For an Abelian category $\mathcal{A}$, the equivalence classes of the objects of $\mathcal{A}$ modulo the relation $[X'] = [X] + [X'']$, when the objects X, X', X'' form a short exact sequence, $0 \longrightarrow X \longrightarrow X' \longrightarrow X'' \longrightarrow 0$, form an Abelian group, called the Grothendieck group, denoted K_0, of $\mathcal{A}$. Here $[X]$ denotes the class of an object in $\mathcal{A}$. The equivalence relation embodies the identification of anti-branes as objects shifted by a unit grade in a complex relative to the objects corresponding to the branes.

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After orientifolding, however, the Grothendieck group falls inadequate to describe the charges of the objects of the Abelian category, $\mathcal{A}$. The charges now are given by the Whitehead group of $\mathcal{A}$, denoted K_1. The classes of objects in the K_1 group carry the information of certain automorphisms of the objects in addition to the objects themselves. Before presenting the definition of the K_1 group, let us note the following from our earlier discussions. We found that the boundary states of the D0-branes are invariant under the orientifolding operation. Further, acting twice, the orientifolding yields the original state back. Thus, given that each boundary state corresponds to an object in the Abelian category $\mathcal{A}$, it is natural to assume the existence of an automorphism, $\mathfrak{f}$ of the objects of $\mathcal{A}$, such that $\mathfrak{f}^2 = 1$. Since an Abelian category is idempotent complete and hence so is its bounded derived category [70], this is a consistent assumption. In order to interpret this automorphism physically, let us note that the CFT analysis points at the stable object on the orientifold backgrounds being a D-instanton which are to be obtained through tachyon condensation of the D0-brane. The latter was found to have tachyonic modes. This bears a close resemblance to the orientifold of the Type-IIB theory, where the orientifold action is a lone $(-1)^{F_L}$ [58, 60, 61]. In this case a D8-brane appears through tachyon condensation of a D9-brane. From this point of view the D8-brane has been interpreted as a pair consisting of a vector bundle V and an automorphism $\alpha : V \longrightarrow V$, where V and α is taken to corresponds to the D9-brane and the tachyon, respectively. In the spirit of this analogy, in the Type-IIA orientifold we can identify the tachyons associated with a D0-brane described by an object X as an automorphism $\mathfrak{f} : X \longrightarrow X$ and D-instantons as a pair $(X, \mathfrak{f})$. Thus, the classes of D-branes relevant for our discussion will carry an extra label designating the automorphism $\mathfrak{f}$. This leads us to consider the Whitehead group K_1 of the Abelian category [59, 60]. The Whitehead group K_1 of the category $\mathcal{A}$ is defined to be an Abelian group generated by equivalence classes $[X, \alpha]$, corresponding to the objects X of $\mathcal{A}$, α being an automorphism of X. The equivalence relations are,

- *Additive:* For any commutative diagram

$$\begin{tikzcd} 0 \arrow[r] & X' \arrow[r] \arrow[d, "\alpha'"'] & X \arrow[r] \arrow[d, "\alpha"'] & X'' \arrow[r] \arrow[d, "\alpha''"'] & 0 \\ 0 \arrow[r] & X' \arrow[r] & X \arrow[r] & X'' \arrow[r] & 0 \end{tikzcd} \tag{6.1}$$

  in $\mathcal{A}$, where the horizontal sequences are exact and the vertical morphisms are automorphisms, we impose an equivalence relation,

$$[X, \alpha] = [X', \alpha'] + [X'', \alpha''] \tag{6.2}$$

- *Multiplicative:* If $\alpha : X \longrightarrow X$ and $\beta : X \longrightarrow X$ are automorphisms in of objects in $\mathcal{A}$, then we impose

$$[X, \alpha\beta] = [X, \alpha] + [X, \beta]. \tag{6.3}$$

Now, for the case at hand, the orientifolding operation lifts up to an automorphism $\mathfrak{f}$ on $\mathcal{A}$, as alluded to above. Then, from equation (6.3) we obtain, $2[X, \mathfrak{f}] = [X, 1]$, which in turn yields

$$[X, \mathfrak{f}] = 0 \mod 2,$$

since, again by equation (6.3), we have $[X, 1] = 0$, which can be seen by setting $\alpha = \beta = 1$. We conclude, therefore, that the Whitehead group K_1 of the category of objects in the orientifold background is isomorphic to $\mathbb{Z}_2$. The $\mathbb{Z}_2$ charge corresponding to $K_1 \sim \mathbb{Z}_2$ is topological and leads to the existence of a single $\mathbb{Z}_2$-charged D-instanton, which we found through the analysis of the boundary states above.

7. Discussions and conclusion

To conclude, we have discussed the boundary states in the spectrum of the Type-IIA orientifold $\mathbb{C}^3/\mathbb{Z}_3 \cdot \Omega \cdot \mathcal{R} \cdot (-1)^{F_L}$. We find that there is no vector multiplet in the closed string massless spectrum. Further, the D0-branes are inflicted with tachyon and become unstable. However, we found the existence of a D-instanton which is stable as the associated tachyons are projected out due to orientifolding.

In spite of the presence of a massless modulus in the closed string spectrum of the twisted NS-NS sector, in this article we refrain from presenting a geometric interpretation of the objects in the aforementioned Abelian category, as the contribution of instantons may develop a superpotential for this modulus [67] rendering the movement over the Kähler moduli space obstructed.

We interpret the D-instanton alluded to above as being obtained through tachyon condensation of the unstable D0-branes. We use the analogy with the mechanism by which D8-branes come into being through tachyon condensation of D9-branes. The associated K-group is given by Whitehead group of the category of D0-branes generated by objects along with an automorphism of the object associated with the tachyon. Contrary to the cases studied earlier in the literature, we assume a lift of the orientifolding on the objects of the Abelian category of D-branes, rather than looking for a geometric realization the target space. One advantage of this description of the K-group is that they are readily generalizable to more exotic orbifolds and, perhaps, to the Calabi-Yau spaces.

However, let us note that the other way to look at the K-group is [58] by considering the fact that the operation $\Omega \cdot \mathcal{R} \cdot (-1)^{F_L}$ corresponds to interchanging (E, F) with $(\overline{F}, \overline{E})$ where E and F are vector bundles and $\overline{E}$ represents the complex conjugate of E. The K-group is given by $KR^1_{\pm}(X)$ [58]. This has not been discussed much in the literature. It will be interesting to compute this and compare with the results here.

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A. Notations & conventions

Here we explain the notations and conventions used in this article and present an inventory of the formulas used. We use light-cone gauge throughout our discussion and double wick-rotated the coordinates, as is customary. Thus, $X^0 \longrightarrow iX^0$ and $X^1 \longrightarrow iX^1$. Similarly $\psi^0 \longrightarrow i\psi^0$ and $\psi^1 \longrightarrow i\psi^1$. The light-cone coordinates are taken to be $X^\pm = X^1 \pm X^2$ and $\psi^\pm = \psi^1 \pm \psi^2$. We impose Dirichlet boundary conditions along the light-cone directions. The *external* directions will be X^0 and X^3; both of which are taken to be euclidean, *i.e.* the external metric is $\delta^{\mu\nu}$. The coordinates $X^4, \ldots, X^9$ will be used to denote the *internal* $\mathbb{C}^3$ directions. We use indices $M, N = 0, \ldots, 9$ for the ten-dimensional spacetime directions and $\widehat{M} = 0, 3, \ldots, 9$ for indices excluding the light-cone directions, $\mu, \nu = 0, 3$ for external indices, $m, n = 4, \ldots, 9$ for internal real indices and $i, \bar{\imath} = 1, 2, 3$ for complexified internal indices. The worldsheet coordinates will be denoted by σ and τ and we use a Euclidean metric on the worldsheet.

A.1 Mode Expansion

A.1.1 Bosonic oscillators

Light cone directions

$$\begin{aligned} X^-(\sigma,\tau) &= x_0^- + \frac{1}{2\pi}P^-\tau + \frac{i}{2}\sum_{l\in\mathbb{Z}}\frac{1}{l}\alpha_l^- e^{-il(\tau+\sigma)} + \frac{i}{2}\sum_{l\in\mathbb{Z}}\frac{1}{l}\widetilde{\alpha}_l^- e^{-il(\tau-\sigma)}, \\ X^+(\sigma,\tau) &= x_0^+ + P^+\tau, \end{aligned} \tag{A.1}$$

the latter implying

$$\alpha_0^+ = \widetilde{\alpha}_0^+ = P^+\delta_{n0}\,, \qquad \alpha_0^- + \widetilde{\alpha}_0^- = P^- \tag{A.2}$$

Untwisted sector

The mode expansion of the Bosonic fields are

$$X^M(\sigma,\tau) = x_0^M + 2P^M\tau + \frac{i}{2}\sum_{l\in\mathbb{Z}}\frac{1}{l}\alpha_l^M e^{-il(\tau+\sigma)} + \frac{i}{2}\sum_{l\in\mathbb{Z}}\frac{1}{l}\widetilde{\alpha}_l^M e^{-il(\tau-\sigma)}, \tag{A.3}$$

For $m = 4, \ldots, 9$, let

$$\frac{1}{\sqrt{2}}(X^{2i+2} + iX^{2i+3}) = Z^i \quad \frac{1}{\sqrt{2}}(P^{2i+2} + iP^{2i+3}) = p^i \qquad i = 1, 2, 3 \tag{A.4}$$

and similarly for right-movers, mutatis mutandis. Thus the mode expansions are given as

$$\begin{aligned} Z^i(\sigma,\tau) &= z_0^i + 2p^i\tau + \frac{i}{2}\sum_{l\in\mathbb{Z}}\frac{1}{l}\alpha_l^i e^{-il(\tau+\sigma)} + \frac{i}{2}\sum_{l\in\mathbb{Z}}\frac{1}{l}\widetilde{\alpha}_l^i e^{-il(\tau-\sigma)}, \\ \overline{Z}^i(\sigma,\tau) &= \overline{z}_0^i + \overline{p}^i\tau + \frac{i}{2}\sum_{l\in\mathbb{Z}}\frac{1}{l}\overline{\alpha}_l^i e^{-il(\tau+\sigma)} + \frac{i}{2}\sum_{l\in\mathbb{Z}}\frac{1}{l}\overline{\widetilde{\alpha}}_l^i e^{-il(\tau-\sigma)} \end{aligned} \tag{A.5}$$

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Twisted sectors

The mode expansions in the twisted sectors, cooresponding to $k = 1, 2$ are

$$\begin{aligned}
X^\mu(\sigma,\tau) &= \frac{i}{2}\sum_{l\in\mathbb{Z}}\frac{1}{l}\alpha_l^\mu e^{-il(\tau+\sigma)} + \frac{i}{2}\sum_{l\in\mathbb{Z}}\frac{1}{l}\widetilde{\alpha}_l^\mu e^{-il(\tau-\sigma)},\\
Z^i(\sigma,\tau) &= \frac{i}{2}\sum_{l\in\mathbb{Z}}\frac{1}{l+kv_i}\alpha^i_{l+kv_i} e^{-i(l+kv_i)(\tau+\sigma)} + \frac{i}{2}\sum_{l\in\mathbb{Z}}\frac{1}{l-kv_i}\widetilde{\alpha}^i_{l-kv_i} e^{-i(l-kv_i)(\tau-\sigma)},\\
\overline{Z}^i(\sigma,\tau) &= \frac{i}{2}\sum_{l\in\mathbb{Z}}\frac{1}{l-kv_i}\overline{\alpha}^i_{l-kv_i} e^{-i(l-kv_i)(\tau+\sigma)} + \frac{i}{2}\sum_{l\in\mathbb{Z}}\frac{1}{l+kv_i}\overline{\widetilde{\alpha}}^i_{l+kv_i} e^{-i(l+kv_i)(\tau-\sigma)}.
\end{aligned} \tag{A.6}$$

A.1.2 Fermionic oscillators

Light-cone directions

$$\psi^+(\sigma,\tau) = \widetilde{\psi}^+(\sigma,\tau) = 0 \quad\Rightarrow\quad \psi^+_r = \widetilde{\psi}^+_r = 0, \tag{A.7}$$

$$\psi^-(\sigma,\tau) = \frac{2}{P^+}\sum_{\widehat{M}=0,3,\dots,9}\psi^{\widehat{M}}\partial_+ X^{\widehat{M}} \quad\Rightarrow\quad \psi^-_r = \frac{1}{P^+}\sum_{\widehat{M}=0,3,\dots,9}\sum_s \alpha^{\widehat{M}}_{r-s}\psi^{\widehat{M}}_s, \tag{A.8}$$

$$\widetilde{\psi}^-(\sigma,\tau) = \frac{2}{P^+}\sum_{\widehat{M}=0,3,\dots,9}\widetilde{\psi}^{\widehat{M}}\partial_- X^{\widehat{M}} \quad\Rightarrow\quad \widetilde{\psi}^-_r = \frac{1}{P^+}\sum_{\widehat{M}=0,3,\dots,9}\sum_s \widetilde{\alpha}^{\widehat{M}}_{r-s}\widetilde{\psi}^{\widehat{M}}_s, \tag{A.9}$$

where $r, s \in \mathbb{Z} + \frac{1}{2}(\mathbb{Z})$ for NS(R)-sector and $\sigma^\pm = \tau \pm \sigma$, $\partial_\pm = \frac{1}{2}(\partial_\tau \pm \partial_\sigma)$.

Untwisted sector

The mode expansions of the left- and right-moving fermions are

$$\psi^M(\sigma,\tau) = \sum_r \psi^M_r e^{-ir(\tau+\sigma)} \quad\text{and}\quad \widetilde{\psi}^M(\sigma,\tau) = \sum_r \widetilde{\psi}^M_r e^{-ir(\tau-\sigma)}, \tag{A.10}$$

respectively, where $r \in \mathbb{Z} + \frac{1}{2}(\mathbb{Z})$ for NS(R)-sectors and $m = 4, \dots, 9$. We define complex fermionic fields,

$$\Psi^i = \frac{1}{\sqrt{2}}(\psi^{2i+2} + i\psi^{2i+3}), \qquad \overline{\Psi^i} = \frac{1}{\sqrt{2}}(\psi^{2i+2} - i\psi^{2i+3}), \tag{A.11}$$

where $i = 1, 2, 3$ and similarly $\widetilde{\Psi}^i$ and $\overline{\widetilde{\Psi}^i}$ for right-movers. The mode expansion for the complexified fermions can be derived from the earlier expressions as

$$\begin{aligned}
\Psi^i(\sigma,\tau) &= \sum_r \Psi^i_r e^{-ir(\tau+\sigma)}, \quad \overline{\Psi^i}(\sigma,\tau) = \sum_r \overline{\Psi^i_r} e^{-ir(\tau+\sigma)}\\
\widetilde{\Psi}^i(\sigma,\tau) &= \sum_r \widetilde{\Psi}^i_r e^{-ir(\tau-\sigma)}, \quad \overline{\widetilde{\Psi}^i}(\sigma,\tau) = \sum_r \overline{\widetilde{\Psi}^i_r} e^{-ir(\tau-\sigma)}
\end{aligned} \tag{A.12}$$

Twisted sectors

The mode expansions of the fermions in the $k=1,2$ twisted sectors are

$$\begin{aligned}
\psi^\mu(\sigma,\tau) &= \sum_r \psi^\mu_r e^{-ir(\tau+\sigma)}, \quad \widetilde{\psi}^\mu_r(\sigma,\tau) = \sum_r \widetilde{\psi}^\mu_r e^{-ir(\tau-\sigma)}, \\
\Psi^i(\sigma,\tau) &= \sum_r \Psi^i_{r+kv_i} e^{-i(r+kv_i)(\tau+\sigma)}, \quad \overline{\Psi^i}(\sigma,\tau) = \sum_r \overline{\Psi^i}_{r-kv_i} e^{-i(r-kv_i)(\tau+\sigma)}, \\
\widetilde{\Psi}^i(\sigma,\tau) &= \sum_r \widetilde{\Psi}^i_{r-kv_i} e^{-i(r-kv_i)(\tau-\sigma)}, \quad \overline{\widetilde{\Psi}^i}(\sigma,\tau) = \sum_r \overline{\widetilde{\Psi}^i}_{r+kv_i} e^{-i(r+kv_i)(\tau-\sigma)}.
\end{aligned} \tag{A.13}$$

Oscillator algebra

We use the following commutators and anti-commutators for the oscillators

$$[\alpha^\mu_l, \alpha^\nu_{l'}] = [\widetilde{\alpha}^\mu_l, \widetilde{\alpha}^\nu_{l'}] = \delta_{l+l',0}\,\delta^{\mu\nu}, \qquad \{\psi^\mu_r, \psi^\nu_s\} = \{\widetilde{\psi}^\mu_r, \widetilde{\psi}^\nu_s\} = \delta_{r+s,0}\,\delta^{\mu\nu}, \tag{A.14}$$

for the external directions and

$$[\alpha^i_{l+kv_i}, \overline{\alpha}^j_{l'-kv_j}] = (l+kv_i)\delta_{l+l',0}\,\delta^{ij}, \quad \{\Psi^i_{r+kv_i}, \overline{\Psi}^j_{s-kv_j}\} = \delta_{r+s,0}\,\delta^{ij}, \tag{A.15}$$

$$[\widetilde{\alpha}^i_{l-kv_i}, \overline{\widetilde{\alpha}}^j_{l'+kv_j}] = (l-kv_i)\delta_{l+l',0}\,\delta^{ij}, \quad \{\widetilde{\Psi}^i_{r-kv_i}, \overline{\widetilde{\Psi}}^j_{r+kv_j}\} = \delta_{r+s,0}\,\delta^{ij}, \tag{A.16}$$

for the internal directions.

A.2 Closed string hamiltonian

The closed string Hamiltonian in the untwisted sector is

$$\begin{aligned}
H^u_c = \pi(P^2+p^2) + 2\pi\Big[&\sum_{\substack{\mu=0,3\\ n\in\mathbb{Z}_+}} (\alpha^\mu_{-n}\alpha^\mu_n + \widetilde{\alpha}^\mu_{-n}\widetilde{\alpha}^\mu_n) + \sum_{\substack{\mu=0,3\\ r>0}} r(\psi^\mu_{-r}\psi^\mu_r + \widetilde{\psi}^\mu_{-r}\widetilde{\psi}^\mu_r) \\
&+ \sum_{\substack{i=1,\ldots,3\\ n\in\mathbb{Z}_+}} (\alpha^i_{-n}\overline{\alpha}^i_n + \widetilde{\alpha}^i_{-n}\overline{\widetilde{\alpha}}^i_n) + \sum_{\substack{i=1,\ldots,3\\ r>0}} r(\Psi^i_{-r}\overline{\Psi}^i_r + \widetilde{\Psi}^i_{-r}\overline{\widetilde{\Psi}}^i_r)\Big] + 2\pi a_u\,,
\end{aligned} \tag{A.17}$$

where $a_u = -1$ in the NS-sector and $a_u = 0$ in the R-R-sector. The fermionic mode-index $r \in \mathbb{Z} + \frac{1}{2}(\mathbb{Z})$ for NS(R)-sector and $\vec{P}$ and $\vec{p}$ are denote the external and internal momenta, respectively. In the twisted sector, on the other hand, the closed string Hamiltonian assumes the form

$$\begin{aligned}
H^T_c = \pi P^2 + 2\pi\Big[&\Big(\sum_{\substack{\mu=0,3\\ n\in\mathbb{Z}_+}} (\alpha^\mu_{-n}\alpha^\mu_n + \widetilde{\alpha}^\mu_{-n}\widetilde{\alpha}^\mu_n) + \sum_{\substack{\mu=0,3\\ r>0}} r(\psi^\mu_{-r}\psi^\mu_r + \widetilde{\psi}^\mu_{-r}\widetilde{\psi}^\mu_r) \\
&+ \sum_{\substack{i=1,\ldots,3\\ n\in\mathbb{Z}}} {}^\circ_\circ(\alpha^i_{n+kv_i}\overline{\alpha}^i_{-n-kv_i} + \widetilde{\alpha}^i_{n-kv_i}\overline{\widetilde{\alpha}}^i_{-n+kv_i}){}^\circ_\circ \\
&+ \sum_{\substack{i=1,\ldots,3\\ r>0}} (r-kv_i)\,{}^\circ_\circ(\Psi^i_{r+kv_i}\overline{\Psi}^i_{-r-kv_i} + \widetilde{\Psi}^i_{r-kv_i}\overline{\widetilde{\Psi}}^i_{-r+kv_i}){}^\circ_\circ\Big] + 2\pi a_T,
\end{aligned} \tag{A.18}$$

where a_T is a constant arising from the normal ordering.

A.3 The ϑ-functions and the f-functions

Let us list the ϑ-functions with characteristics for convenience [69],

$$\vartheta \begin{bmatrix} \alpha \\ \beta \end{bmatrix} \left(\nu \middle| \tau\right) = e^{2\pi i\alpha\beta} \widehat{q}^{\alpha^2/2-1/24}\ \eta(\tau)\ \prod_{n=1}^{\infty} \left(1+\widehat{q}^{n-1/2+\alpha} e^{2\pi i(\beta+\nu)}\right) \left(1+\widehat{q}^{n-1/2-\alpha} e^{-2\pi i(\beta+\nu)}\right), \tag{A.19}$$

where $\widehat{q} = e^{2\pi i\tau}$ and one needs to choose $\alpha, \beta \in (-1/2, 1/2]$. Here $\eta(\tau)$ is the Dedekind η-function.

$$\eta(\tau) = \widehat{q}^{1/24} \prod_{n=1}^{\infty} (1-\widehat{q}^n). \tag{A.20}$$

The Jacobi ϑ-functions are given by the following ϑ-functions with characteristics

$$\begin{aligned} \vartheta \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} (\nu|\tau) &= \vartheta_1(\nu|\tau), \qquad & \vartheta \begin{bmatrix} \frac{1}{2} \\ 0 \end{bmatrix} (\nu|\tau) &= \vartheta_2(\nu|\tau), \\ \vartheta \begin{bmatrix} 0 \\ 0 \end{bmatrix} (\nu|\tau) &= \vartheta_3(\nu|\tau), \qquad & \vartheta \begin{bmatrix} 0 \\ \frac{1}{2} \end{bmatrix} (\nu|\tau) &= \vartheta_4(\nu|\tau). \end{aligned} \tag{A.21}$$

We also use the following f-functions, related to the ϑ-functions [53, 50]

$$f_1(\widehat{q}) = \widehat{q}^{1/12} \prod_{n=1}^{\infty} (1-\widehat{q}^{2n}), \qquad f_2(\widehat{q}) = \sqrt{2}\widehat{q}^{1/12} \prod_{n=1}^{\infty} (1+\widehat{q}^{2n}), \tag{A.22}$$

$$f_3(\widehat{q}) = \widehat{q}^{-1/24} \prod_{n=1}^{\infty} (1+\widehat{q}^{2n-1}), \quad f_4(\widehat{q}) = \widehat{q}^{-1/24} \prod_{n=1}^{\infty} (1-\widehat{q}^{2n-1}). \tag{A.23}$$

A.4 Modular S transformation

The behavior of the Dedekind's η-function and the Jacobi ϑ-functions under the modular S transformation is given by,

$$\eta\left(-\frac{1}{\tau}\right) = (-i\tau)^{-1/2}\eta(\tau), \tag{A.24}$$

$$\vartheta_1\left(\frac{\nu}{\tau}| -\frac{1}{\tau}\right) = -(-i\tau)^{1/2} e^{\pi i\nu^2/\tau}\vartheta_1(\nu|\tau), \tag{A.25}$$

$$\vartheta_2\left(\frac{\nu}{\tau}| -\frac{1}{\tau}\right) = (-i\tau)^{1/2} e^{\pi i\nu^2/\tau}\vartheta_4(\nu|\tau), \tag{A.26}$$

$$\vartheta_3\left(\frac{\nu}{\tau}| -\frac{1}{\tau}\right) = (-i\tau)^{1/2} e^{\pi i\nu^2/\tau}\vartheta_3(\nu|\tau), \tag{A.27}$$

$$\vartheta_4\left(\frac{\nu}{\tau}| -\frac{1}{\tau}\right) = (-i\tau)^{1/2} e^{\pi i\nu^2/\tau}\vartheta_2(\nu|\tau). \tag{A.28}$$

The modular S transformation for the f-functions for a real (used for annulus) and an imaginary (used for Möbius) arguments are,

A.4.1 Real arguments

$$\begin{aligned} f_1(e^{-\pi s}) &= \frac{1}{\sqrt{s}} f_1(e^{-\pi/s}), \qquad & f_2(e^{-\pi s}) &= f_4(e^{-\pi/s}), \\ f_3(e^{-\pi s}) &= f_3(e^{-\pi/s}), \qquad & f_4(e^{-\pi s}) &= f_2(e^{-\pi/s}). \end{aligned} \tag{A.29}$$

A.4.2 Imaginary arguments

$$\begin{aligned} f_1(ie^{-\pi s}) &= \frac{1}{\sqrt{2}s} f_1(ie^{-\pi/4s}), \qquad & f_2(ie^{-\pi s}) &= f_2(ie^{-\pi/4s}), \\ f_3(ie^{-\pi s}) &= e^{i\pi/8} f_4(ie^{-\pi/4s}), \qquad & f_4(ie^{-\pi s}) &= e^{-i\pi/8} f_3(ie^{-\pi/4s}). \end{aligned} \tag{A.30}$$

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